國 立 清 華 大 學 命 題 紙

八十八學年度 電機工程 系（所） 乙 組碩士班研究生招生考試

科目 工程數學 科號 4501 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. (10%) A fair die is successively rolled. (The outcome of one roll of a fair die is 1, 2, 3, 4, 5, or 6, each with probability 1/6.) Let X and Y denote, respectively, the number of rolls necessary to obtain a 3 and a 4. Find (a) $E[X]$ and (b) $E[X|Y=3]$.

2. (20%) Let X be a normal random variable with mean 0 and variance 1 and let I, independent of X, be such that $P\{I=1\} = P\{I=0\} = 1/2$. Now define Y by

$$Y = \begin{cases} X, & \text{if } I = 1 \\ -X, & \text{if } I = 0. \end{cases}$$

In words, Y is equally likely to equal either X or $-X$.

(a) Are X and Y independent? Why?

(b) Show that Y is normal with mean 0 and variance 1.

(c) Show that the covariance $\mathrm{Cov}(X,Y) = 0$.

(d) Do (a), (b), and (c) contradict the fact that uncorrelated jointly normal random variables are independent?

3. (10%) Let

$$A = \begin{pmatrix} 2 & -3 \\ 2 & -5 \end{pmatrix}.$$

Find e^A.

4. (10%) Let

$$A = \begin{pmatrix} -2 & 1 & -1 \\ 0 & 2 & 1 \\ -4 & 2 & 2 \\ 0 & 4 & 0 \end{pmatrix}$$

and

$$b = \begin{pmatrix} -1 \\ 1 \\ 1 \\ -2 \end{pmatrix}$$

Find a vector p such that p is in the column space of A and $b - p$ is orthogonal to every vector in the column space of A.

5. (10%) Let

$$A = \begin{pmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix}.$$

Use the Gram-Schmidt process to obtain an orthonormal basis for the column space of A.

6. (10%) Let $F(z)$ and $G(z)$ be two functions of complex varaible z as follows:

$$F(z) = |z|^2$$

$$G(z) = \frac{z^2}{z - 0.5}$$

(a) Is $F(z)$ differentiable? Why?

(b) $G(z)$ can be expressed as the following Laurent series expansion

$$G(z) = \sum_{n=-\infty}^{\infty} g(n) z^{-n}$$

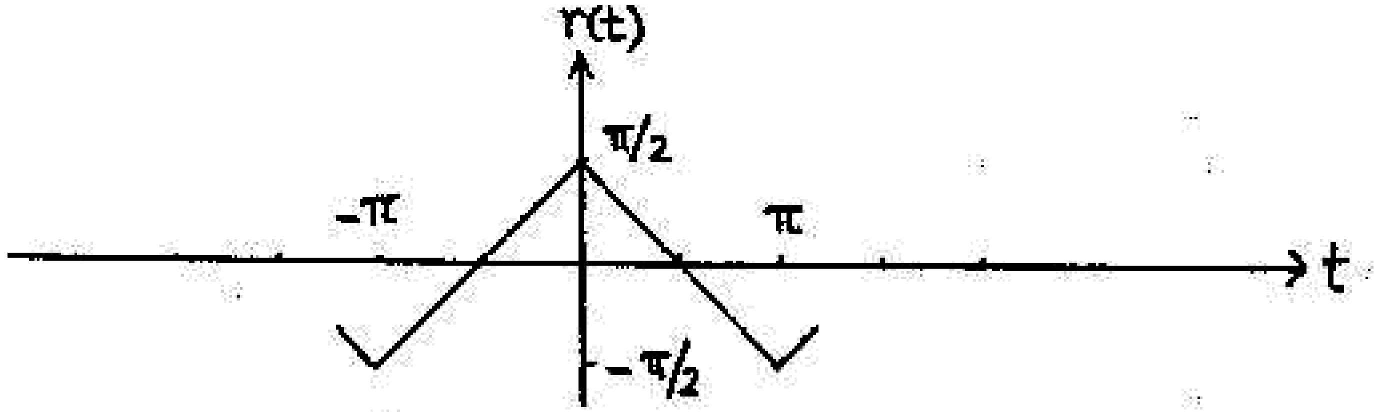
for $|z| > 0.5$. Find $g(n)$.

7. (10%) Solve the following first-order differential equation

$$(e^y + x)\frac{dy}{dx} = 1$$

8. (20%) Let $r(t)$ be a periodic triangle function with period equal to 2π as shown below. Over $[-\pi, \pi]$ (one period), $r(t)$ is given by

$$r(t) = \begin{cases} t + \frac{\pi}{2}, & -\pi \leqslant t < 0 \\ -t + \frac{\pi}{2}, & 0 < t \leqslant \pi \end{cases}$$



(a) Find the Fourier series of $r(t)$.

(b) Based on the results of part (a), please find the steady state solution $Y(t)$ of the following differential equation

$$\frac{d^2Y(t)}{dt^2} + 0.02\frac{dY(t)}{dt} + 25Y(t) = r(t)$$