國 立 清 華 大 學 命 題 紙

八十七學年度 電機工程 系（所） 丙 組碩士班研究生入學考試

科目 近代物理 科號 3104 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. A neutral pion whose kinetic energy is equal to its rest energy decays into two gamma-ray photons in flight. Find the angle between these two gamma-ray photons that are produced if their energies are the same. (Hint: Use the relativistic expression for the kinetic energy to find the momentum of the pion.) (20%)

2. Consider the quantum mechanics of a particle with mass m moving in the following one-dimensional potential

$$V(x)=\begin{cases}\infty & \textbf{when } x\le 0\\ 0 & \textbf{when } 0<x\le \mathbf{d}\\ V_0 & \textbf{when } x>\mathbf{d}\end{cases}$$

(a) What is the (unnormalized) wave function for $0<x\le \mathbf{d}$ if the particle has an energy $E<V_0$? (5%)

(b) What is the (unnormalized) wave function for $x>\mathbf{d}$ if the particle has an energy $E<V_0$? (5%)

(c) Derive the equation satisfied by the energy eigenvalues of the bound states. (10%)

3. Consider the quantum mechanics of a particle with mass m moving in the following one-dimensional triangular potential

$$V(x)=\begin{cases}-kx & \textbf{when } x\le 0\\ kx & \textbf{when } x>0\end{cases}$$

Estimate the ground state energy with the following steps.

(a) Let (-d, d) be the range of the space where the electron moves. Estimate the kinetic energy of the particle with the uncertainty principle. (5%)

(b) Write down the total energy of the particle, which is the sum of the kinetic energy and potential energy. (5%)

(c) Minimize the total energy with respect to d. What is the ground state energy? (10%)

4 qualitative plot

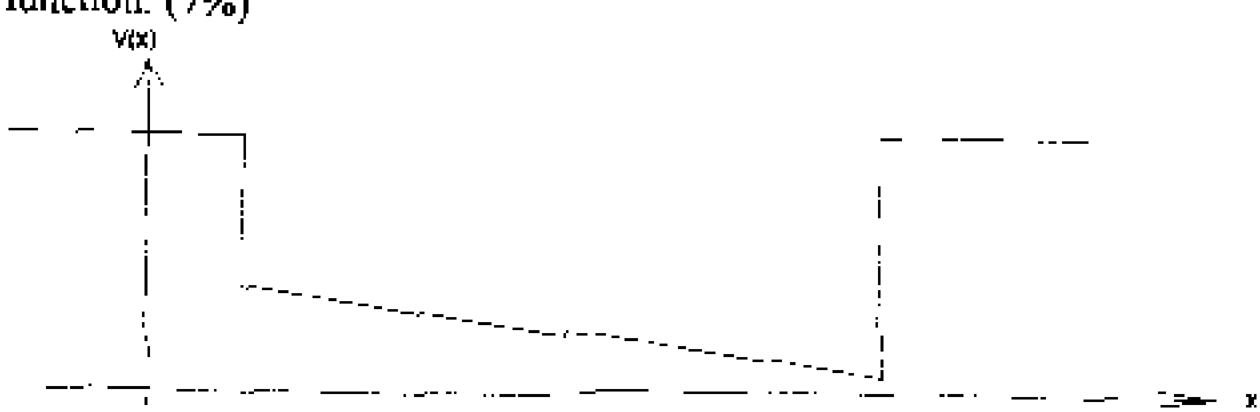
(a), Consider a particle moves in a potential well V(x), where

$$V(x)=\begin{cases} V_1 & \text{when } x > L/2 \\ 0 & \text{when } L/2 > x > -L/2 \\ V_0 & \text{when } x < -L/2 \end{cases} \qquad (V_0 > V_1 > 0)$$

Plot qualitatively the wave function corresponding to the second energy level in the potential well. Note carefully the number and the relative position (with respect to the origin) of the nodes, as well as the slope of the wave function at $x=\pm L/2$. (5%)

(b). As in (a), since the potential within the well vanishes, the wave function within the well may be written as sinusoidal functions, i.e., sin or cos. One can therefore talk about the wavelength λ of the wave function in the well. Compare first the qualitative plot you draw for the second energy level in (a) with that of the second energy level in an infinite well of well width L; find the lower limit of λ for the second energy level. Secondly, while still satisfying the required slope at $x=\pm L/2$, increase λ as much as possible in your qualitative plot to find the upper limit of λ for the second energy level. From these, find the upper and lower bound of this second energy level. (8%)

(c) Consider now the potential well as shown in the following figure. Plot qualitatively the wave function corresponding to the fifth energy level in this potential well. Note carefully the number as well as the spacing of the nodes, and the amplitude of the wave function. (7%)



5. One-sided simple harmonic oscillator