國 立 清 華 大 學 命 題 紙

八十七學年度 電機工程 系(所) 丙 組碩士班研究生入學考試

科目 工程數学 科號 3101 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. (a) Write a first-order linear ordinary differential equation whose solution is
   $6t + e^{-t}$. (6%)

   (b) Write a third-order linear ordinary differential equation whose solution is again $6t + e^{-t}$. (7%)

2. The complex integral $\oint_C \frac{(z-b)(z-c)\,dz}{(z-a)^2}$ is equal to zero, where a, b and c are undetermined complex numbers, and C denotes a contour enclosing the point $z = a$.

   Find a, b and c. You should consider all possibilities for these numbers.

   (Note that these numbers may or may not be the same, and the contour C may or may not enclose $z = b$ and $z = c$.) (18%)

3. Find the d'Alembert's solutions of the following partial differential equations.

   (a) $u_{xx} + 2u_{xy} + u_{yy} = 0$. (5%)

   (b) $xu_{xy} = yu_{yy} + u_y$. (6%)

4. Let A be an n x n matrix with characteristic polynomial

   $f(t) = (-1)^n t^n + a_{n-1}t^{n-1} + \cdots + a_1 t + a_0$.

   The characteristic polynomial is defined as $f(t) = \det(A - tI)$, where I is an n x n unit matrix.

   (a) Find the value of $\det(cA)$ in terms of $a_{n-1}, a_{n-2}, \ldots$ and a_0, where c is constant. (8%)

   (b) Find the value of tr(A) in terms of $a_{n-1}, a_{n-2}, \ldots$, and a_0. (10%)

5. Find the surface integral of the vector function

   $\mathbf{F} = y^3\,\mathbf{i} + x^3\,\mathbf{j} + z^3\,\mathbf{k}$

   over the portion of the surface defined as

   S: $x^2 + 4y^2 = 4$, $x \ge 0$, $y \ge 0$, $0 \le z \le h$. (12%)

6. G(w) is the Fourier Transform of a real function g(x). We have

$$G(w)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(x)e^{-jwx}\,dx.$$

Now a real function f(x) is related to g(x) as

$$f(x)=\frac{2}{3}g(x)+\frac{1}{4}[g(x-a)+g(x+a)],$$

where a is a positive real number, and the Fourier Transform of f(x) is F(w). Answer the following questions:

(A). What is the relationship between F(w) and G(w)? (8%)

(B). If f(x) is known, can one obtain g(x) accordingly? If the answer is yes, how? (8%)

7. Solve the following initila value problem by means of Laplace Transform.

$$\begin{cases} y_1' = 6y_1 + 9y_2 \\ y_2' = y_1 + 6y_2 \end{cases}, \quad y_1(0)=-3,\ y_2(0)=-3. \qquad (12\%)$$