國 立 清 華 大 學 命 題 紙

八十七學年度 電機工程 系 (所) 乙 組碩士班研究生入學考試

科目 訊號與系統 科號 3003 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. Let

$$p_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

where $\delta(t)$ is the unit impulse (or Dirac delta) function, $T > 0$ is a constant, and

$$x(t) = \text{sinc}\left(\frac{t}{2T}\right)\cos\left(\frac{\pi t}{T}\right)$$

with

$$\text{sinc}(t) = \begin{cases} \sin(\pi t)/(\pi t), & t \neq 0 \\ 1, & t = 0 \end{cases}$$

(a) Find the Fourier transform (spectrum), $X(\omega)$, of $x(t)$. (5%)

(b) Find the spectrum, $X_p(\omega)$, of $x_p(t) = x(t)p_T(t)$. Is it possible to recover $x(t)$ from $x_p(t)$? Why? (10%)

(c) If $p_T(t)$ is passed through a continuous-time linear time-invariant filter whose impulse response is $h(t) = x(t)$, find the output $y(t)$ of the filter. (5%)

(You need write down detailed derivations, otherwise no credits)

2. Consider a continuous-time linear time-invariant system with impulse response given by $h(t) = te^{-\alpha t}u(t)$, where $\alpha > 0$ and $u(t)$ is the unit step function.

(a) Find the transfer function, $H(s)$, of the system. Is the system causal stable? (Why?) (5%)

(b) Find the step response of the system. (5%)

(You need write down detailed derivations, otherwise no credits)

3. Let $h[n]$ be the impulse response of a discrete-time linear time-invariant system with transfer function given by

$$H(z) = \frac{z^{-1}}{(1 - 0.5z^{-1})(1 - 2z^{-1})}$$

It is known that $h[n]$ is a symmetric sequence (i.e., $h[n] = h[-n]$).

(a) What is the region of convergence of $H(z)$ and what is $h[n]$? (5%)

(b) Find the output $y[n]$ of the system in response to the input $x[n] = u[n] \cdot w[n]$ as $n \to \infty$, where

$$w[n] = \begin{cases} 2, & n \text{ is even} \\ 0, & n \text{ is odd} \end{cases}$$

and $u[n]$ is the unit step sequence. (5%)

(You need write down detailed derivations, otherwise no credits)

4. Consider a continuous-time linear time-invariant system with impulse response given by $h(t)=\sin \pi t[u(t)-u(t-3)]$, where $u(t)$ is the unit step function. Find and sketch the output signal $y(t)$ for each of the following input signals $x(t)$:

   (a) $x(t)=\sum_{k=-\infty}^{\infty}\delta(t-2k)$, where $\delta(t)$ is the unit impulse function. (5%)

   (b) $x(t)=u(t)-u(t-2)$. (5%)

5. Compute and sketch the output signal $y[n]$ for each of the following discrete-time linear time-invariant systems with input signal $x[n]$ and impulse response $h[n]$:

   (a) $x[n]=u[n]-u[n-n_1-1]$ and $h[n]=u[n]-u[n-n_2-1]$, where $u[n]$ is the unit step sequence and $n_2>n_1>0$. (5%)

   (b) $x[n]=a^n u[n]$ and $h[n]=b^n u[n]$, where $a\neq b$ and $0<a,b<1$. (5%)

6. Consider a linear time-invariant causal digital filter that has the following observed causal input and output sequences of finite length:

time n:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
input $x[n]$:	1	1	2	3	1	2	4	0	0	0	0	0	0	0	0	0
output $y[n]$:	1	3	6	9	10	9	9	10	9	1	−3	2	4	0	0	0

   (a) Determine the length of the system impulse response sequence based on the observed data. (2%)

   (b) Determine the values of the first four points of the system impulse response sequence. (8%)

7. Consider a sequence $x[n]$ whose discrete-time Fourier transform has the following property:

$$X(e^{j\Omega})=\begin{cases}1, & |\Omega|<\pi/6\\ 0.5, & \pi/6<|\Omega|<\pi/3\\ 0, & \pi/3<|\Omega|<\pi\end{cases}.$$

   (a) Define a new sequence $y[n]$ with values $y[n]=x[3n], n=0,\pm1,\pm2,\pm3,\ldots$ Express the discrete-time Fourier transform $Y(e^{j\Omega})$ of $y[n]$ in terms of $X(e^{j\Omega})$, and then sketch it. (6%)

   (b) Define another new sequence $z[n]$ with values $z[n]=y[n/3]$ for $n=0,\pm3,\pm6,\pm9,\ldots$ and $z[n]=0$ otherwise. Express the discrete-time Fourier transform $Z(e^{j\Omega})$ of $z[n]$ in terms of $Y(e^{j\Omega})$, and then sketch it. (6%)

   (c) Is it possible to recover the original sequence $x[n]$ from $z[n]$? Why? (3%)

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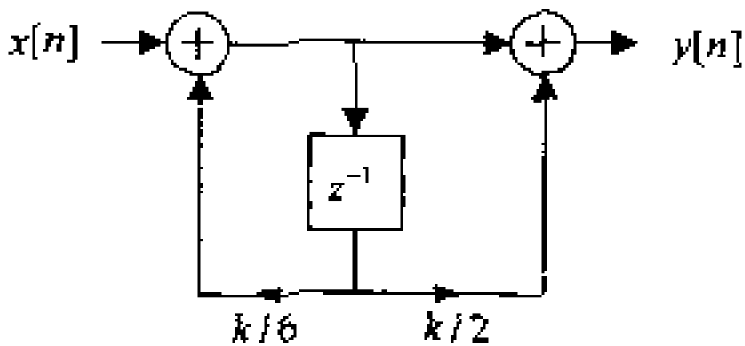
8. Consider the following linear causal digital filter structure:



In this structure, z^{-1} denotes a unit delay.

(a) Find the system function $H(z)$, plot the pole-zero diagram, and indicate the region of convergence. (5%)

(b) Determine the range of values of k such that the system and its inverse system are both causal and stable. Draw the direct-form-II structure of the corresponding inverse system. (5%)

(c) Determine $y[n]$ if $k=1$ and $x[n]=1+2^{-n}+3^{n}$ for all n. (5%)