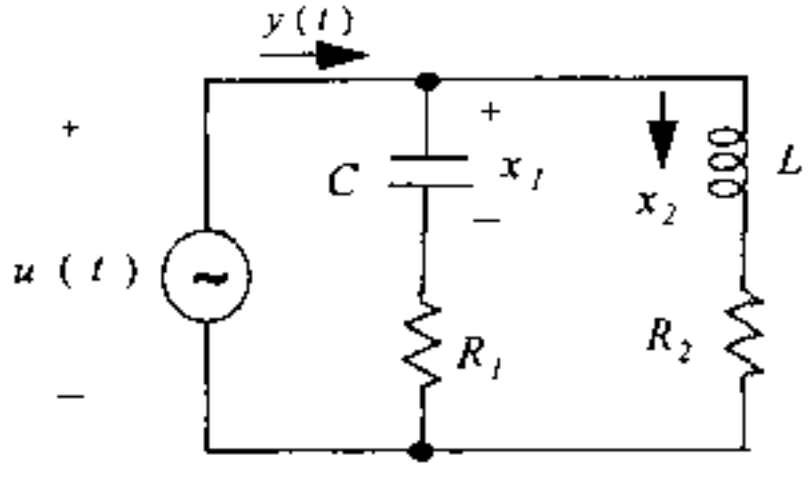
1. Consider the following circuit



(a) using the capacitor voltage and inductor current as state variables, write state equation and output equation for the system.

(b) Find the conditions relating R_1, R_2, C and L that render the system uncontrollable. Find similar conditions for unobservable system. (20%)

2. The equations of motion for a satellite are

$$\begin{cases} \ddot{x} - 2\omega\dot{y} - 3\omega^2 x = 0 \\ \ddot{y} + 2\omega\dot{x} = u, \quad where \quad \omega = 2\pi / (3600 \times 24) rad / \sec. \end{cases}$$

Choose $X = [x, \dot{x}, y, \dot{y}]^T$ as the state vector and y as the measurement, and design a full-order observer with poles placed at $s = -2\omega, -3\omega$ and $-3\omega \pm 3j\omega$ (20%)

3. (a) Describe the root-locus method briefly for single input single output linear time invariant feedback systems by using an example. (10%)

(b) Give one approach to solving the following polynominal equation by adapting the above root locus concept:

$$P(s) = s^5 + 4s^4 + 3s^3 + 2s^2 + s + 1 = 0$$

In other words, you are required to formulate the above root-finding problem into a root-locus plotting problem to obtain the five roots. You do not need to give the numerical results except the solution steps. Also, the answer is not unique. (10%)

4. Given the system :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

the closed-loop system is implemented by the following state feedback control:

$$u(t) = -[g_1, g_2, g_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

with g_1, g_2 and g_3 being real constants. Determine the constraints on the elements of g_1, g_2 and g_3 so that the overall system is asymptotically stable. (10%)

5. (a) The controller as shown is to be realized using OP amplifiers, draw the circuit and show all component values. (15%)

(b) It is known that the pure differentiator in the feedforward controller may lead to control problem in practical implementation , briefly describe the problem and give your approach to solve it. (5%)

(c) If this controller is to be implemented digitally with sampling interval T=0.1s, find the control algorithm m_n , where m_n denotes the control output at n-th sampling interval. (10%)