八十八學年度 電 機(物)系(所) 丙 組碩士班研究生入學考試

科目 近代物理 科號3204 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. (a) Assuming that hydrogen atom has a circular electron orbit and the centripetal force and the electric force are balanced, calculate the total energy of the hydrogen atom using classical dynamics. (7%)

(b) Bohr's condition for hydrogen atom is that an electron orbit contains an integral number of de Broglie wavelengths. Find the orbital radius of hydrogen atom in this Bohr model. (8%)

2. An electron is confined in a one-dimensional box with side L. (a) Calculate the energy of the electron. (b) If the ground state energy of the electron is equal to kT at 300K, what is the value of L? (7% + 8%)

3. Two particles of mass m_1 and m_2 respectively interact with each other in an otherwise free space. Let $\mathbf{p}_1, \mathbf{p}_2$ be the momenta, and $\mathbf{r}_1, \mathbf{r}_2$ be the position vectors (relative to the origin) of the two particles in some frame of reference (S). The Hamiltonian of the two-particle system is given by $H = \frac{\mathbf{p}_1^2}{2m_1} + \frac{\mathbf{p}_2^2}{2m_2} + \frac{K(\mathbf{r}_1 - \mathbf{r}_2)^2}{2}$.

(a) Introducing new variables $\mathbf{R} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$, $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$ (the center-of-mass position vector and the total momentum) and $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $\mathbf{p} = \frac{m_2\mathbf{p}_1 - m_1\mathbf{p}_2}{m_1 + m_2}$ (the relative position vector and momentum), express the Hamiltonian H in term of these new variables (Hint: H should be given in the form $H(\mathbf{R},\mathbf{P}) + H(\mathbf{r},\mathbf{p})$, where $H(\mathbf{R},\mathbf{P})$ contains only $\mathbf{R},\mathbf{P}$, and $H(\mathbf{r},\mathbf{p})$, only $\mathbf{r},\mathbf{p}$) (7%)

(b) Starting from the commutator relationship in the frame of reference S, show that the part $H(\mathbf{R},\mathbf{P})$ and $H(\mathbf{r},\mathbf{p})$ in your expression of H in (a) commute, that is., $[H(\mathbf{R},\mathbf{P}), H(\mathbf{r},\mathbf{p})]=0$. (6%)

(c) Now solve the time-independent Schrodinger equation of the two-particle system and find the energy and eigenstates in frame of reference S. (7%)

八十六學年度 電機(电子) 系(所) 丙 組碩士班研究生入學考試

科目 近代物理 科號 324 共 2 頁第 2 頁 *請在試卷【答案卷】內作答

4. A particle of mass m moves one-dimensionally in a potential $V(x) = V_0 \tan^2(\frac{x}{a})$. Suppose we are only interested in the ground state, that is, the movement of the particle close to the origin, find the approximate value of the ground state energy. (15%)

5. Wavefunctions and symmetry.

i) Consider a Hydrogen atom. Plot qualitatively the wave function $\psi_{1S}(r)$, where r is the distance of the electron from the nucleus. (5%)

ii) Consider a ring of four coupled H-atoms. Suppose that the system has a 4-fold rotational symmetry, and that there is only one state (e.g. 1S) on each isolated atom. Plot qualitatively the wavefunction of the lowest energy eigenstate along the ring. (10%)

6. Chemical bond.

i) Explain why normally the energy of an electronic state increases with its number of nodes. (10%)

ii) Consider an H_2^{+1} molecule (with one plus charge). Explain how the formation of chemical bond lowers the kinetic energy of electrons. (10%)