國立清華大學命題紙

八十六學年度 電機所丙組 暨電子所 組碩士班研究生入學考試

科目 工程數學 科號 3201 共 二 頁第 1 頁 *請在試卷【答案卷】內作答

1. Solve (a) $x(x-1)y'' - xy' + y = 0$. (10%)

   (b) $\frac{d}{dt}\mathbf{y} = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}\mathbf{y} + \begin{bmatrix} -6 \\ 2 \end{bmatrix}e^{-2t}$ (10%)

2. f(t) is a periodic function of period 2π of which its Fourier Series exists and, for $-\pi \le t \le \pi$, is given as follows:

$$f(t) = \begin{cases} 1, & -\pi/2 \le t \le \pi/2 \\ 0, & -\pi \le t < -\pi/2 \text{ and } \pi/2 < t \le \pi \end{cases}$$

(a). Write f(t) in terms of its Fourier Series expansion.(5%)

(b). g(t) is also a periodic function of period 2π of which its Fourier Series exists and, for $-\pi \le t \le \pi$, is given as follows

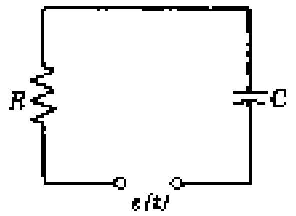
$$g(t) = \begin{cases} 1, & -\pi/4 \le t \le 3\pi/4 \\ 0, & -\pi \le t < -\pi/4 \text{ and } 3/4\,\pi < t \le \pi, \end{cases}$$

How are its Fourier Coefficients related to those of the above f(t) ? (10%)

3. (a) Find the Laplace transform of the function $e(t)$:

$$e(t) = \begin{cases} 10t & \text{volts if } 0<t<4, \\ 40 & \text{volts if } t>4 \end{cases} \qquad (5\%)$$

(b) Find the current $i(t)$ in the following RC-circuit, where R = 10 ohms, C = 0.1 farad, and the initial charge on the capacitor is 0. The applied voltage source $e(t)$ is as given in the above (a). (10%)



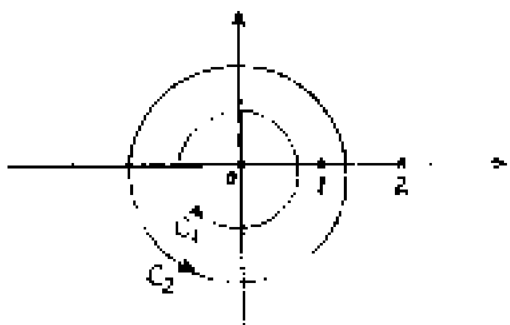
八十六學年度 電機所丙組暨電子所 組碩士班研究生入學考試

科目 工程數學 科號 3201 共 二 頁第 2 頁 *請在試卷【答案卷】內作答

4 A vector field $\mathbf{F} = y^3\mathbf{i} + x^3\mathbf{j} + z^3\mathbf{k}$. Evaluate surface integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dA$, where $S: x^2 + 9y^2 = 9,\ x \ge 0,\ y \ge 0$, and $0 < z < 5$. (15%)

5 Let the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. Find A^n for any positive n. (15%)

6. Compute the integrals $\oint_{C_1} \dfrac{-2z+3}{z^2-3z+2}\,dz$ and $\oint_{C_2} \dfrac{-2z+3}{z^2-3z+2}\,dz$, where C_1 and C_2 are the counterclockwise contours(i.e., circles centered at $z = 0$) as shown below. (10%)



7. Compute the integral $\displaystyle\int_0^{2\pi} \frac{1}{2+\sin x}\,dx$. (10%)