國立清華大學命題紙

電子工程

八十五學年度 電機 系（所） 丙 組碩士班研究生入學考試

科目 近代物理 科號 3104 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Based on the uncertainty principle $\Delta x \Delta p > \hbar/2$,

a) calculate the minimum average kinetic energy of a particle with mass m confined in a limited region of space Δx (10%)

b) estimate the minimum kinetic energy of a particle with mass 0.2 Kg confined in a box of size 0.3 meter. (5%)

2. Consider the following step potential

$$V(x) = 0, \quad x < 0$$
$$V(x) = \infty, \quad x > 0$$

Suppose a particle with energy E (> 0) is incident from $-\infty$.

a) Derive the wave function of this particle for both $x < 0$ and $x > 0$. (Normalization is not required.) (10%)

b) Calculate the probability current carried by this particle. (5%)

c) Calculate the reflection coefficient. (5%)

3. Consider a particle moving inside the potential well with

$$V(x) = 0, \; -L < x < L$$
$$V(x) = \infty, \; |x| > L$$

Assume the particle is in the ground state.

a) Calculate the wave function. (5%)

b) Calculate the probability density P(p), with P(p)dp the probability that one finds the particle's momentum lying inside the interval (p, p−dp), where dp is a small differential. (10%)

4. Experiments indicate that 13.6 eV is required to separate a hydrogen atom into a proton and an electron, that is, its total energy is E= − 13.6 eV. Using classical dynamics, calculate the orbital radius and velocity of the electron in a hydrogen atom. (15%)

5.
(a) A particle of mass m moves in a potential V(x). Suppose that V(x) is bounded below; that is, there exists V_0 such at $V(x) \geq V_0$. Show that the energy E of the particle is also bounded below by V_0. (15%)
(b) For an arbitrary bounded wave function $\phi(x)$, define a functional

$$\varepsilon[\phi(x)] = \frac{\int \phi^*(x)[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)]\phi(x)dx}{\int \phi^*(x)\phi(x)dx} \text{ or } \frac{\langle\phi|-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)|\phi\rangle}{\langle\phi|\phi\rangle}$$ in the bra-ket

notation. Assume again now V(x) is bounded below. Prove that $\varepsilon[\phi(x)]$ finds its minimum E when $\phi(x)$ is the ground state wave function of the Schrodinger equation

$[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)]\phi(x) = E\phi(x)$. (Hint: expand $\phi(x)$ into the sum of eigenstates.)
(Note. This is the Rayleigh principle.) (10%)
(c) Take any convenient trial function $\phi(x)$, and apply the Rayleigh principle described in (b) to get an estimate of the ground state energy for the case $V(x) = A|x|$, where A is a positive number. (10%)