國 立 清 華 大 學 命 題 紙

八十五學年度 電機(電子)工程系(所) 丙 組碩士班研究生入學考試

科目 工程數學 科號 3101 共 三 頁第 1 頁 *請在試卷【答案卷】內作答

1. Solve the following ordinary differential equations.

   a) $y'''(x) - 3y''(x) + 3y'(x) - 1 = 0$; $y(0) = 0$, $y'(0) = 0$, $y''(0) = 2$. (5%)

   b) Solve for $Y(t)$ which satisfies

$$X'(t) = Y(t)$$
$$Y'(t) = -X(t) + 2Y(t)$$

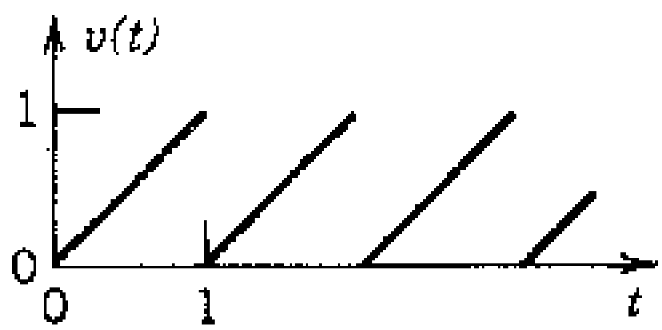
   with $Y(0) = 0$, $Y'(0) = 1$. (5%)

2. Find the Fourier series of the following function with a period of 2π

$$f(x) = \begin{cases} (\pi^2 + \pi x)/2 & , \text{if } -\pi < x < 0 \\ -\pi x/2 & , \text{if } 0 < x < \pi \end{cases}$$ (10%)

3. (a) Find the Laplace transform of the periodic saw-tooth function $v(t)$ as shown in the following figure (7%).

   (b) Determine the current $i(t)$ in a single loop L-R-C circuit when L=1 henry, R=3 ohms, and C=0.5 farad, $i(0)=1$ amp, $v_C(0)=0$ volt, and the applied voltage source is as given in the above question (a). (8%)



4. Find the general solution of the one-dimensional wave equation

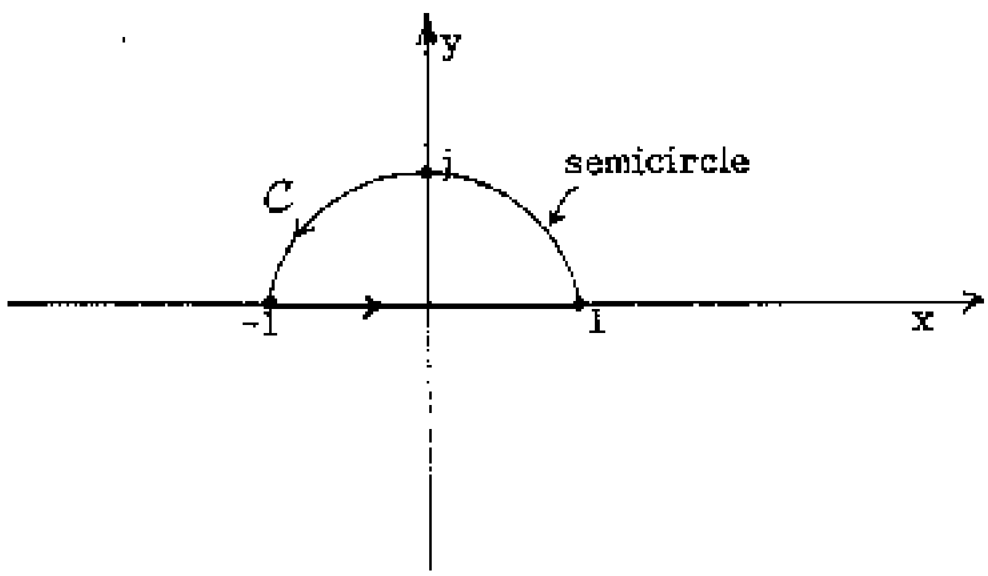
$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t),$$

   subject to the boundary condition $\psi(x=0,t) = \psi(x=L,t) = 0$. (15%)

5. Assume a complex function $z(t)$ satisfies the differential equation $dz/dt + jz = 0$ and the condition $z(0)=3$, where t is real and j is equal to $\sqrt{-1}$. Find $z(t)$. (10%)

6. (a) Express $\tan z$ as $\sum_{n=0}^{\infty} a_n (z-\frac{\pi}{4})^n$ in some region. Here you only have to find a_0, a_1, and a_2. (8%)

(b) Compute $\int_C \tan z \, dz$ where C is a counterclockwise contour as shown below. (7%)



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$$D=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(A). Find all the eigenvalues of D. (4%)

(B). Find the eigenvector of D corresponding to the largest eigenvalue. (4%)

(C). Give the set of $\underline{a}$'s such that $D\underline{a} = 0$. (4%)

8. (A). It is known that the real system of linear equations $A\underline{x} = b$, where A is an NxN matrix, has a unique solution. Then what is the rank of A ? Which real number can not be A's eigenvalue ? (5%)

(B). Consider the 3x3 system of linear equations:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ \alpha \end{bmatrix} \qquad (1)$$

If $\alpha=1$, how many solutions does it have ? Give the reason.(4%)

(C). Consider the matrix equation as in (1). Give the α's that will make this matrix equation having no solution? Explain your answer.(4%)