國立清華大學命題紙

八十五學年度 電機工程 系（所） 乙 組碩士班研究生入學考試

科目 訊號與系統 科號 3003 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. When the input $x(n)$ of a discrete-time linear-time invariant (LTI) system $H(z)$ is

$$x(n) = (1/2)^n u(n) + 2^n u(-n-1)$$

where $u(n)$ is a unit-step sequence, the output is given by

$$y(n) = 6(1/2)^n u(n) - 6(3/4)^n u(n)$$

(a) Find the impulse response $h(n)$ of the system for all values of n. (5%)

(b) Determine the steady state response $y(n)$ for large n, when the input is given by $x(n) = [1 + cos(\pi n + 0.2\pi)]u(n)$. (5%)

(c) Find the impulse response of the stable inverse system of $H(z)$. (5%)

(You need to write down all the derivations in detail)

2. Assume that $x(n)$ is a discrete-time signal and $X(e^{j\Omega})$ is its Fourier transform.

(a) Write the expressions of the relationship between $x(n)$ and $X(e^{j\Omega})$. Prove one of them. What are the physical meanings of these expressions? Is $X(e^{j\Omega})$ a continuous periodic function of Ω? (5%)

(b) Find the inverse Fourier transform, denoted $x'(n)$, of $Re\{X(e^{j\Omega})\}$ (real part of $X(e^{j\Omega})$). Does $x'(n)$ have any symmetry properties? If yes, what are they ? (5%)

(c) Assume that $\tilde{X}(k) = X(e^{j\Omega})$ for $\Omega = 2\pi k/N$ and

$$\tilde{x}(n) = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}(k) e^{j2\pi kn/N}$$

(inverse DFT of $\tilde{X}(k)$). What is the relationship between $\tilde{x}(n)$ and $x(n)$? (5%)

(You need to write down all the derivations in detail)

3. The magnitude squared of the frequency response of an Nth-order continuous-time lowpass Butterworth filter (causal stable) is given by

$$|B_N(j\omega)|^2 = \frac{1}{1 + (\omega/\omega_c)^{2N}}$$

(a) What is the transfer function $B_2(s)$? (5%)

(b) What is the steady state response $y(t)$ for large t, when the input of $B_2(s)$ is given by

$$x(t) = sin(\omega_c t)u(t)$$

where $u(t)$ is a unit-step function? (5%)

(You need to write down all the derivations in detail)

4. Consider a discrete-time LTI system with impulse response $h(n) = a^n u(n)$, where $0 < a < 0.5$.
   (a) Compute and plot the unit-step response $s(n)$ of the system. (6%)
   (b) Compute and plot the output $y(n)$ of the system when the input is $x(n) = u(n+2) - 2u(n-3) + u(n-8)$. (6%)

5. Consider an invertible continuous-time LTI system with impulse response $h(t)$ and frequency response $H(j\omega)$, where the corresponding inverse system has impulse response $h_I(t)$ and frequency response $H_I(j\omega)$.
   (a) Write down the relationship between $h(t)$ and $h_I(t)$. Also, express $H_I(j\omega)$ in terms of $H(j\omega)$. (3%)
   (b) If $h(t) = \delta(t) - e^{-2t}u(t)$, find $h_I(t)$. (4%)
   (c) Let echoes occur every T seconds apart in a satellite communication system, and assume that the corresponding transmission channel can be modeled by a continuous-time LTI system with impulse response

$$h(t) = \sum_{k=0}^{\infty} c^k \delta(t - kT)$$

   where c is a constant with $|c|<1$. Find the frequency response of $h(t)$. Also, determine the impulse response and frequency response of the ideal echo canceller for the satellite communication system. (6%)

6. A sampling process can be modeled as the multiplication of the input signal $x(t)$ by a uniform impulse train

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

   where T is the sampling period. Let $X(j\omega)$ be the input spectrum.
   (a) Show that the corresponding output spectrum is

$$Y(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X[j(\omega - k\omega_s)]$$

   with $\omega_s = 2\pi / T$. (8%)
   (b) If $X(j\omega)$ is bandlimited with $|\omega| \le \omega_b$, find the condition for the sampled signal to be aliasing-free. (3%)
   (c) What is the physical interpretation of this sampling theorem on a pure sinusoidal signal? (2%)
   (d) Show how to recover $x(t)$ from the sampled signal. (2%)

7. (*Single-Sideband Amplitude-Modulated Discrete-Time Signals*)

A discrete-time message sequence $m(k)$ has a low-pass spectrum

$$M\left(e^{j\Omega}\right) = \sum_{k=-\infty}^{\infty} m(k)\, e^{-j\Omega k}$$

This sequence is amplitude-modulated to form $x(k) = m(k)\cos\Omega_c k$.

(a) Please express the spectrum of $x(k)$, $X(e^{j\Omega})$, in terms of $M\left(e^{j\Omega}\right)$. (2%)

(b) An ideal Hilbert transformer is defined as

$$H\left(e^{j\Omega}\right) = \begin{cases} -j, & 0 \le \Omega < \pi \\ j, & -\pi \le \Omega < 0 \end{cases}$$

Please find the corresponding impulse response $h(n)$. (5%)

(c) Is $h(n)$ realizable and stable? If yes, prove it. If no, state the reason and state the way to approximately implement it. (3%)

(d) Let $y(k) = m(k) + j\hat{m}(k)$, where $\hat{m}(k)$ is the Hilbert transform of $m(k)$. Please find its Fourier transform $Y\left(e^{j\Omega}\right)$ in terms of $M\left(e^{j\Omega}\right)$ and $H\left(e^{j\Omega}\right)$. (2%)

(e) From (d), find $Y\left(e^{j\Omega}\right)$ in terms of $M\left(e^{j\Omega}\right)$. (2%)

(f) An upper-sideband amplitude-modulated signal is

$$x_U(k) = m(k)\cos\Omega_c k - \hat{m}(k)\sin\Omega_c k$$

Please find the spectrum $X_U\left(e^{j\Omega}\right)$ of $x_U(k)$. (6%)