八十五學年度 電機工程 系（所） 乙 組碩士班研究生入學考試
科目 工程數學 科號 3001 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. (10%) Let R be the set of all real numbers. Let V be the vector space of all polynomials over R. Please show that V is infinite-dimensional.

2. (10%) Let V be a subspace of R^4 with an orthogonal basis $\{[-1,-1,1,1],[2,-1,0,1]\}$. Please find the orthogonal complement $V^\perp$ of V, i.e.

$$V^\perp = \{\vec{w} \in R^4 | \vec{w} \cdot \vec{v} = 0,\ \forall \vec{v} \in V\}.$$

Let $\vec{u} = [3,-1,0,-2]$ be a vector in R^4. Please find a vector $\vec{v}$ in V and a vector $\vec{w}$ in $V^\perp$ such that $\vec{u} = \vec{v} + \vec{w}$.

3. The Fourier transform $X(j\omega)$ of a continuous-time signal $x(t)$ is defined as

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Find $X(j\omega)$ for the following signals:
(a) $x(t) = sin(\omega_0 t + \pi/7)$ where $\omega_0 > 0$. (5%)
(b)

$$x(t) = \int_{-\infty}^{\infty} h(\tau) h(\tau + t) d\tau$$

where

$$h(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & otherwise \end{cases}$$

(5%)
(c) $x(t) = \sum_{k=-\infty}^{\infty} \delta(t + kT)$ where $T > 0$ and $\delta(t)$ is the Dirac delta function. (5%)
(Please write down detailed derivations, otherwise no credit)

4. The Laplace transform $X(s)$ of a continuous-time signal $x(t)$ is defined as

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

for all s belonging to Region of Convergence (ROC).
(a) Find $X(s)$ for $x(t) = e^{-b|t|}$ and the associated ROC where $b > 0$. (5%)
(b) Find the output $y(t)$ of the following second-order differential equation system

$$y''(t) + 3y'(t) + 2y(t) = x'(t) - x(t)$$

when the input $x(t) = \delta(t)$ (the Dirac delta function) provided that the initial conditions $y'(0) = y(0) = 0$. (5%)
(c) Find the output $y(t)$ of the differential equation system given in Part (b) when the input $x(t)$ is given by

$$x(t) = 5u(t - 100)$$

where $u(t)$ is the unit step function. (5%)
(Please write down detailed derivations, otherwise no credit)

5. Suppose that we have a coin and two boxs which are labeled as #A and #B respectively. Box #A contains 3 red balls, 2 white balls, and 1 blue ball; while box #B contains 2 red balls and 4 white balls. We then throw the coin. If "head" is shown, we randomly pick up a ball from box #A; otherwise, we randomly pick up a ball from box #B. Assume that the probability of obtaining "head" is P(H)=0.6: (10%)

(a) Please write down the outcome space of this experiment. How many different events exist in that outcome space?

(b) Suppose that the "head" is shown, and then a red ball is picked up. How many different events have occurred?

(c) Randomly find someone to do the experiment. What is the probability that he will pick up a red ball?

(d) Suppose that somebody get a red ball in the experiment. What is the probability that he get this ball from box #B?

6. Suppose that X and Y are jointly normal random variables, and both are zero mean. The conditional density function

$$f(y \mid x) = \frac{1}{\sigma_y \sqrt{2\pi(1-\gamma^2)}} \exp\left\{ - \frac{(y - \gamma \sigma_y \cdot x/\sigma_x)^2}{2\sigma_y^2 (1-\gamma^2)} \right\}$$

(10%)

(a) Find the regression line $\Phi(x)$ that minimizes the mean square error.

(b) Show that the random variables X and $Z = (Y - \Phi(X))$ are indepentdent.

7. True or false. You should give reasons or counterexamples, otherwise no credits.

(a) (4%) Define the sum $V + W$ of two subspaces V and W of a real vector space to be the set

$$V + W = \{\vec{v} + \vec{w} \mid \vec{v} \in V, \vec{w} \in W\}.$$

Then, this set is a subspace too.

(b) (3%) If the rank of an $m \times n$ matrix is 4, it is impossible that there are two linearly dependent columns in this matrix.

(c) (3%) A is a 5×5 matrix with three eigenvalues λ_1, λ_2 and λ_3. Both of the eigenspaces E_1 and E_2 corresponding to λ_1 and λ_2 have dimension 2. Then A is diagonalizable.

8. Evaluate the following integral, (10%)

$$J = \oint_c \left(\frac{ze^{\pi z}}{z^4 - 16} + ze^{\pi/z} \right) dz$$

Where C is the ellipse $9x^2 + y^2 = 9$ counterclockwise.

9. Using the method of least squares, fit a straight line y=a+bx to the following four points. (10%)

(-1.0, 1.00), (-0.1, 1.1), (0.2, 0.8), (1.0, 1.0)