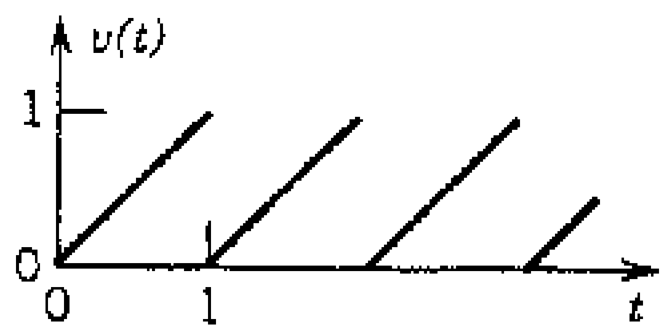
1. Find the Fourier series of the following function with a period of 2π

$$f(x)=\begin{cases}(\pi^2+\pi x)/2 & ,\text{ if } -\pi<x<0\\ -\pi x/2 & ,\text{ if } 0<x<\pi\end{cases}\qquad (10\%)$$

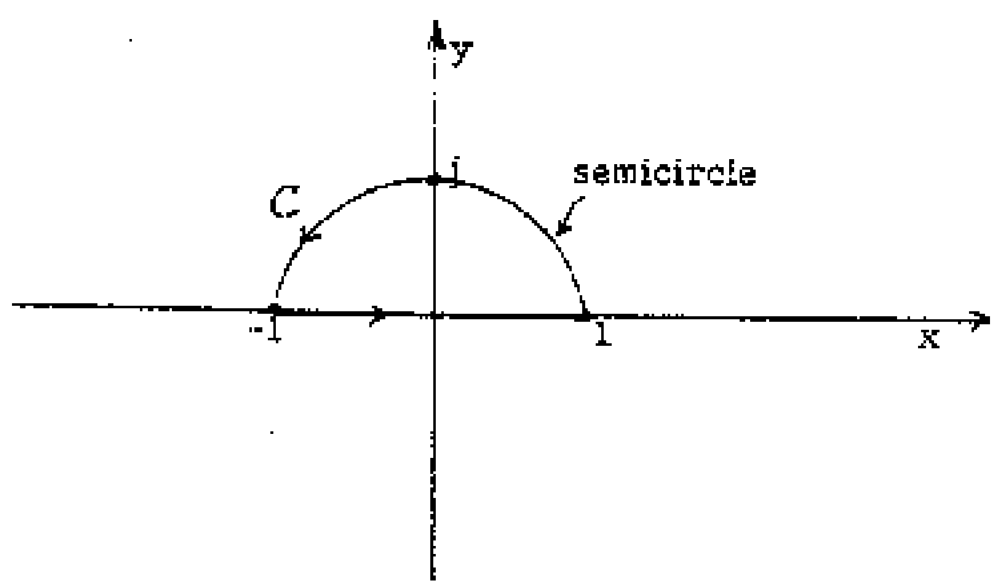
2. (a) Find the Laplace transform of the periodic saw-tooth function $v(t)$ as shown in the following figure (7%).

(b) Determine the current $i(t)$ in a single loop L-R-C circuit when L=1 henry, R=3 ohms, and C=0.5 farad, $i(0)=1$ amp, $v_C(0)=0$ volt, and the applied voltage source is as given in the above question (a). (8%)



3. (a) Express $\tan z$ as $\sum_{n=0}^{\infty} a_n\left(z-\frac{\pi}{4}\right)^n$ in some region. Here you only have to find a_0, a_1, and a_2. (8%)

(b) Compute $\int_C \tan z\, dz$ where C is a counterclockwise contour as shown below. (7%)



4. The differential equation for the deflection y of a beam, which rests on an elastic foundation and is subjected to a concentrated load P at $x = x_0$ from one end is

$$\frac{d^4 y}{dx^4} + 4b^4 y = \frac{P}{EI}\delta(x - x_0)$$

where $4b^4$ accounts for the elasticity effect, E is the Young's modulus, I is the second moment of the cross-section of the beam, and b, E, I, P are constants.
If y(x) and dy/dx = 0 at x = 0 and x = L, and $d^2y/dx^2 = 0$ at x = 0, find the deflection y(x).

(15%)

5. (a) What is the Caley-Hamilton Theorem?

(b) If f(x) is an analytic scalar function of a scalar x, show that for an n x n matrix A

$$f(\mathbf{A}) = r_1\mathbf{A}^{n-1} + r_2\mathbf{A}^{n-2} + \dots + r_{n-1}\mathbf{A} + r_n\mathbf{I},$$

where I is the n x n identity matrix.

(c) Find cosA for A $= \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

(15%)

6. (A). It is known that the real system of linear equations $A\underline{x} = b$, where A is an NxN matrix, has a unique solution. Then what is the rank of A? Which real number can not be A's eigenvalue? (5%)

(B). Consider the 3x3 system of linear equations:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ \alpha \end{bmatrix} \qquad (1)$$

If $\alpha = 1$, how many solutions does it have? Give the reason.(5%)

(C). Consider the matrix equation as in (1). Give the α's that will make this matrix equation having no solution? Explain your answer.(5%)

7.

$$D=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(A). Find all the eigenvalues of D. (5%)

(B). Find the eigenvector of D corresponding to the largest eigenvalue. (5%)

(C). Give the set of $\underline{a}$'s such that $D\underline{a} = 0$. (5%)