國 立 清 華 大 學 命 題 紙

八十四學年度 電機工程 所 丙 組碩士班研究生入學考試

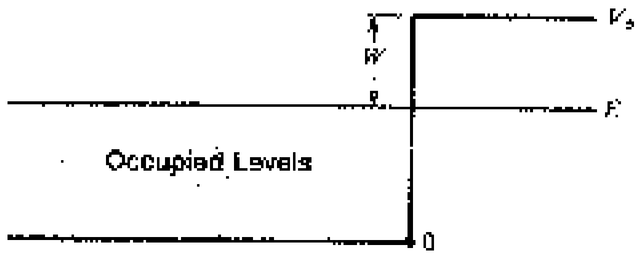
科目 近代物理 科號 2404 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

(1) The density of free electrons in metal is about 10^{22} to $10^{23}\ cm^{-3}$. The interface between the metal and the air may be model as a potential step as shown in the figure.

(a) **EXPLAIN** why this step potential sounds reasonable. (5%)

(b) If an extremely sharp positively charged needle is placed a few Å away from the metal. This step potential model is no longer valid. **GIVE** a new potential model to reflect the effect of this charged needle. (5%)

(c) **DISCUSS** the behavior of those energetic electrons in the metal under the influence of this charged needle. (5%)



(2) Consider a one dimensional square potential barrier of height V_0 and thickness a (as shown in *Fig. 2a*). The corresponding transmission coefficient for this square potential barrier as a function of $\frac{E}{V_0}$ is plotted in *Fig. 2b*. (where $\frac{mV_0a^2}{\hbar^2} = 8$ is used)

(a) **EXPLAIN** why the transmission coefficient is an oscillating instead of a monotone increasing function. (5%)

(b) The extremum of the transmission coefficient occurs at some particular E_n. After some algebra, a relation for this perfect transmission coefficient is found as

$$E_n = (nC)^b$$

where C is a constant. Without doing any algebra, EXPLAIN why the above expression holds. In addition, DETERMINE the value of index b. (5%)

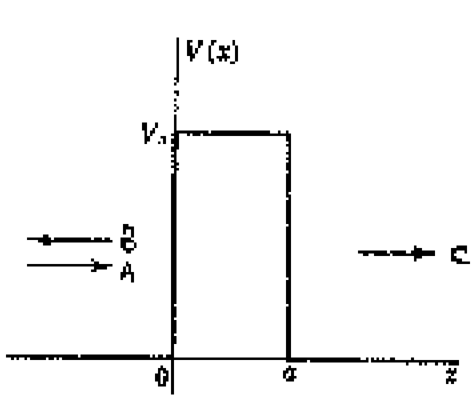


Fig. 2a)

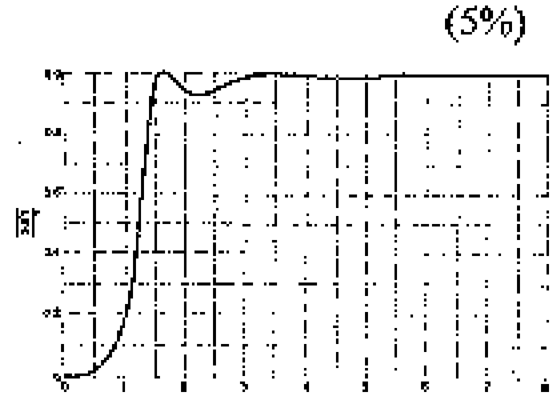


Fig. 2b

(3) A particle of mass m moves in a one dimensional potential $V(x) = -V_0\delta(x)$, where V_0 is a positive real constant.

(a) Starting from the Schrodinger equation, **FIND** the continuity conditions at $x = 0$. (5%)

(b) **FIND** the bound state energy. (5%)

(4) (a) **PROVE** that the bound states in a one dimensional potential cannot be degenerate. (Hint: Assume the potential is $V(x)$ and f and g are degenerate bound states, Find the Wronskian $W(f,g) = f\frac{dg}{dx} - g\frac{df}{dx}$. Does this depend on x?) (10%)

(b) By using the result of (a), **SHOW** that the wavefunction of the bound states in a one dimensional potential is always real. (5%)

(5) If an arbitrary initial state function for a particle in a one dimensional box is expanded in the discrete series of eigenstates of the Hamiltonian relevant to this one dimensional box, one obtains

$$\Psi(x,0) = \sum_{n=1}^{\infty} b_n(0)\varphi_n(x)$$

On the other hand, if the particle is free, its Hamiltonian has a continuous spectrum of eigenenergies and an arbitrary initial state become an integral of eigenstate φ_k as follows:

$$\Psi(x,0) = \int_{-\infty}^{\infty} b(k)\varphi_k dk$$

(a) **WHAT** are the dimensions of $|b_n|^2$ and $|b(k)|^2$, respectively? (5%)

(b) **WHAT** are the source of the difference in the dimensionality? (5%)

(c) **WHAT** are the dimensions and the physical interpretations of the following integral:

$$\int_{-\infty}^{\infty} |b(k)|^2 dk \ ?$$

(5%)

(d) **WRITE** down the wavefunction $\Psi(x,t)$ for the above two initial states.

(5%)

(6) Suppose an electron is confined in a one dimensional box of length L.

(a) **SOLVE** for the wavefunctions of the ground state $\Psi_0(x,t)$, the first excited state $\Psi_1(x,t)$, and the second excited state $\Psi_2(x,t)$. (10%)

(b) Suppose the electron is in the mixed state $\Psi(x,t) = a\Psi_0 + b\Psi_1$. **CALCULATE** the expectation value of its dipole moment, which oscillates in time. **WHAT** is the frequency of the dipole? (8%)

(c) Suppose the electron is in the mixed state $\Psi(x,t) = c\Psi_0 + d\Psi_2$. **CALCULATE** the expectation value of its dipole moment. **WHAT** does it tell you about the electronic transition from the second excited state to the ground state? (7%)

(7) **WHAT** is the expectation of momentum for an electron propagating in Bloch wavefunction with spatial

$$\Psi(x) = e^{jkx}u(x)$$

where $u(x)$ is a periodic function. (5%)