國立清華大學命題紙

八十四學年度 電机工程研究所 丙 組碩士班研究生入學考試

科目 電磁學 科號 2403 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

Some Useful Vector Identities

$$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = \mathbf{C}\cdot\mathbf{A}\times\mathbf{B}$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$$

$$\nabla(\psi V) = \psi\nabla V + V\nabla\psi$$

$$\nabla\cdot(\psi\mathbf{A}) = \psi\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla\psi$$

$$\nabla\times(\psi\mathbf{A}) = \psi\nabla\times\mathbf{A} + \nabla\psi\times\mathbf{A}$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$

$$\nabla\cdot\nabla V = \nabla^2 V,$$

$$\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$

$$\nabla\times\nabla V = 0$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$

$$\int_V \nabla\cdot\mathbf{A}\,dv = \oint_S \mathbf{A}\cdot d\mathbf{s} \qquad \text{(Divergence theorem)}$$

$$\int_S \nabla\times\mathbf{A}\cdot d\mathbf{s} = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell} \qquad \text{(Stokes's theorem)}$$

Gradient, Divergence, Curl, and Laplacian Operations

Cartesian Coordinates (x, y, z)

$$\nabla V = \mathbf{a}_x\frac{\partial V}{\partial x} + \mathbf{a}_y\frac{\partial V}{\partial y} + \mathbf{a}_z\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates (r, ϕ, z)

$$\nabla V = \mathbf{a}_r\frac{\partial V}{\partial r} + \mathbf{a}_\phi\frac{\partial V}{r\,\partial\phi} + \mathbf{a}_z\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial A_\phi}{r\,\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \frac{1}{r}\begin{vmatrix} \mathbf{a}_r & \mathbf{a}_\phi r & \mathbf{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (R, θ, ϕ)

$$\nabla V = \mathbf{a}_R\frac{\partial V}{\partial R} + \mathbf{a}_\theta\frac{\partial V}{R\,\partial\theta} + \mathbf{a}_\phi\frac{1}{R\sin\theta}\frac{\partial V}{\partial\phi}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix} = \mathbf{a}_R\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right] + \mathbf{a}_\theta\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial\phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \mathbf{a}_\phi\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial\theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

1. Please derive the expression of the curl of a vector $\vec{E}$ in the generalized coordinate system (12%)

$$\nabla\times\vec{E}=\begin{vmatrix}\dfrac{\mathbf{a}_1}{h_2h_3} & \dfrac{\mathbf{a}_2}{h_3h_1} & \dfrac{\mathbf{a}_3}{h_1h_2}\\ \dfrac{\partial}{\partial u_1} & \dfrac{\partial}{\partial u_2} & \dfrac{\partial}{\partial u_3}\\ h_1E_1 & h_2E_2 & h_3E_3\end{vmatrix}$$

where h_1, h_2, and h_3 are metric constants.

2. The potential within a hydrogen atom is given by

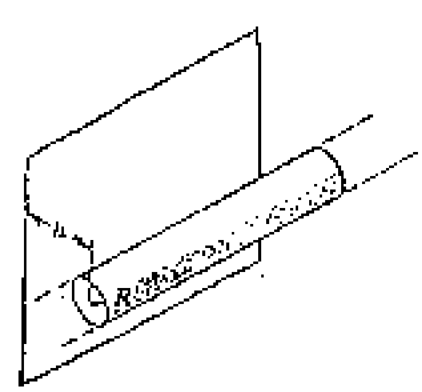
$$V(r)=\frac{Ae^{-\lambda r}}{r}\left(1+\frac{\lambda r}{2}\right)$$

where λ and A are constants. Please find the corresponding charge distribution responsible for this potential. Exclude the origin from your calculations and assume a perfect sphere for a hydrogen atom. (5%)

3. A parallel-plate capacitor consists of two parallel metallic plates separated by a uniform distance d. The space between the two plates is filled with a linear dielectric material of a constant permittivity ε. Neglecting the fringing effect, find quantitatively and sketch qualitatively the distribution of polarization charge induced in the capacitor when connected to a d-c voltage source of V_0. (10%)

4. A circular long wire having radius R is set parallel to a plane conductor. The distance between the center of the wire and the plane is h. Please derive the capacitor per unit length (F/m) is as (14%)

$$C=\frac{2\pi\varepsilon}{\ln\left[\dfrac{h}{R}+\sqrt{\left(\dfrac{h}{R}\right)^2-1}\right]}\qquad (in\ F/m)$$



Hint: Use the method of image.

5. We consider field intensities of electromagnetic waves in this problem.

(i) The required minimum electric field intensity for an AM radio station is 25 mV/m. What is the power density associated with this minimum field? What is the corresponding minimum magnetic field intensity H? (5%)

(ii) The electric field as high as 2×10^{11} V/m is achieved by a pulse Nd-glass laser. What is the power density (in W/m^2) associated with this field? What is the corresponding magnetic field intensity H? (5%)

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6. Do the following phase representations of free-space electric fields satisfy the Maxwell's equations? If so, find the associated magnetic field. If not, please explain why? (all in the rectangular coordinate)

(i) $\vec{E} = \hat{x}\ E_0\, e^{-jkx}$ (3%)

(ii) $\vec{E} = \hat{y}\ E_0 \sin(\frac{\pi x}{a})\, e^{-jkz}$ $\quad 0 \le x \le a,\ 0 \le y \le b$ (8%)

7. For the two distributions of magnetic vector potential $\vec{A}$ given below, find the associated sources in free space.

(i) $\hat{z}\ \dfrac{e^{-jk_0 r}}{r}$

(ii) $\hat{z}\ e^{-jk_0|x|}$

where k_0 is the propagation constant in free space and r is the radial distance in the spherical coordinate. (14%)

8. An air-filled waveguide has inside dimensionals of $4cm \times 2cm$. Along such a waveguide, the measured wavelength λ_g of a mode is 6 cm. Which is this propagation mode? Determine the associated phase and group velocities of this mode. (12%)

9. Let us consider guided electromagnetic waves propagating in different communication channels.

(i) Please schematically plot the attenuation of electromagnetic waves versus frequency in the case of co-axial transmission lines. Please explain your results. (4%)

(ii) Please schematically plot the attenuation of electromagnetic waves versus frequency in the case of rectangular metallic waveguides. Please explain your results. (4%)

10. Ten personal computers are connected to a workstation through coaxial cables. The bit rate of transmission of data among these computers is 100 MHz. The whole network was designed well and the performance of the system is beyond a certain goal, say the error rate << 10^{-9}. One personal computer is disconnected and taken away for other usages. No extra arrangement is made and its connecting cable is left in the system alone. Please discuss the communication or received signals between any personal computer and workstation will be. Please explain possible reasons as much you can. (4%)