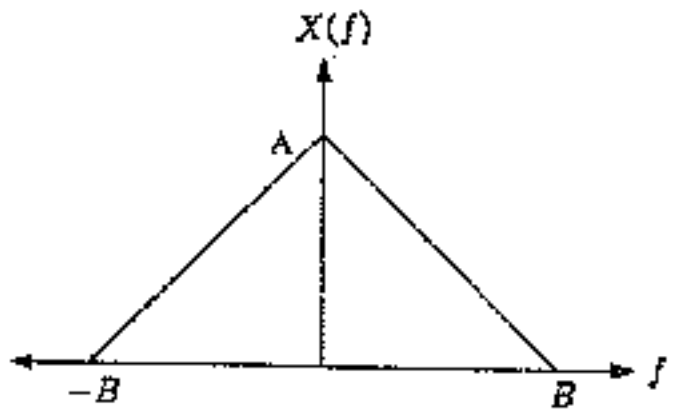
1. Consider an analog signal $x(t)$ with spectrum $X(f)$ shown below.

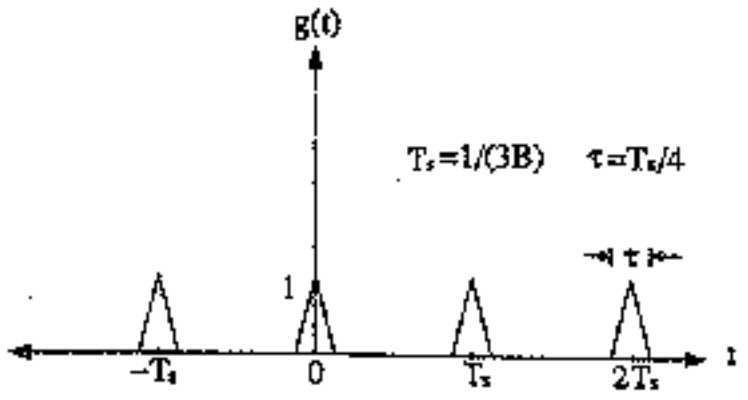


(a) Is the following statement true? Why? (4%)
"$x(t)$ is specified by at most $2B$ independent pieces of information per second."

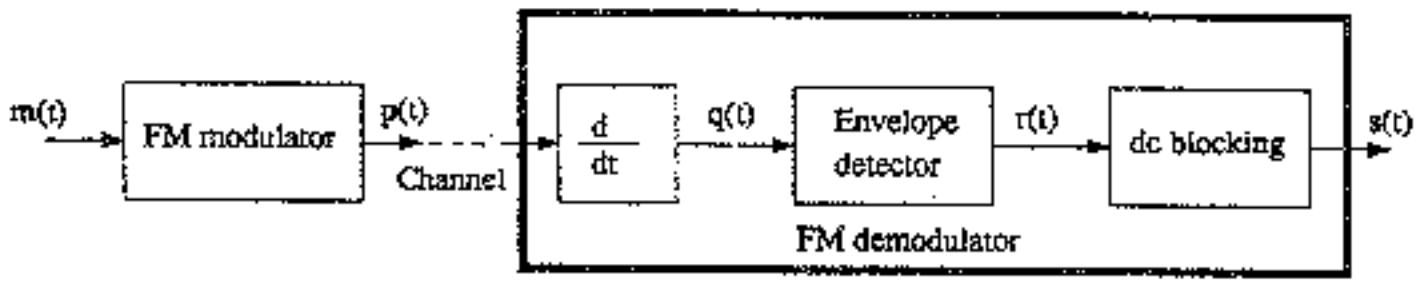
(b) If $x(t)$ is sampled (or multiplied) by a periodic unit impulse train and then uniformly quantized into $L = 2^n$ levels, determine the minimum channel bandwidth required for binary pulse code modulation to transmit the sampled signal without interference. (3%)

(c) If $x(t)$ is sampled (or multiplied) by a periodic unit impulse train with a period of $1/B$ second, sketch the spectrum of the sampled signal. (3%)

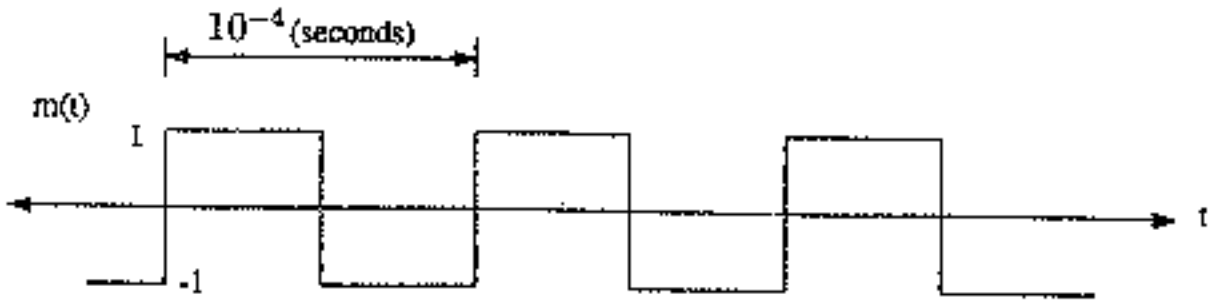
(d) If $x(t)$ is sampled (or multiplied) by the function shown below, sketch the spectrum of the sampled signal for $|f| \leq 4B$. Is it possible to reconstruct $x(t)$ from the sampled signal? Why? (10%)



2. Consider the frequency modulation (FM) system shown below, where $m_p = m(t)_{\max} = |m(t)_{\min}|$, the carrier frequency is f_c, the carrier amplitude is A, and the maximum deviation of the carrier frequency is $\Delta f = (k_f/2\pi) \times m_p$.



(a) If $f_c = 100$ MHz, $k_f = \pi \times 10^4$, and $m(t)$ is given below, sketch the waveforms of $p(t)$, $q(t)$, $r(t)$, and $s(t)$ without considering the channel noise. (8%)

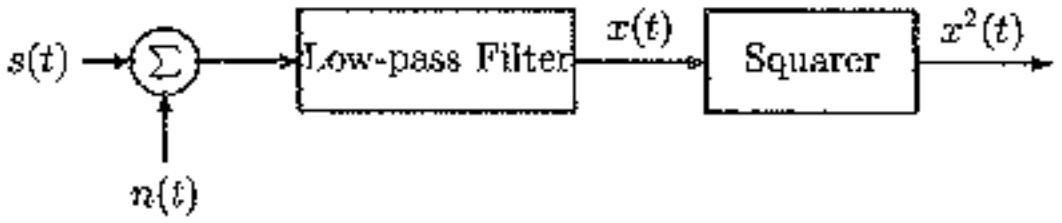


(b) Determine the transmission bandwidth (i.e., B_{FM}) for the case given in (a). (4%)

(c) If $m(t) = \cos^3(10,000t)$ and $k_f = 471,000$, determine the corresponding transmission bandwidth. (4%)

(d) If the carrier amplitude A is not a constant, but a function of time, modify the FM demodulator properly such that the effect of the carrier amplitude variations can be alleviated. (4%)

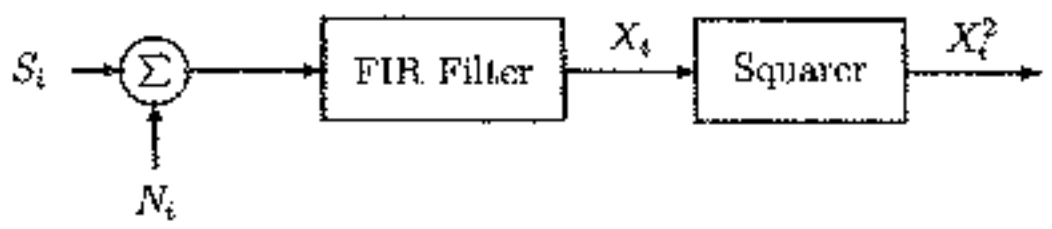
3. Consider the following model of a communication system:



where $s(t)$ is a baseband real signal with bandwidth W, $n(t)$ is a white Gaussian noise with two-sided power spectral density $N_0/2$, the low-pass filter has bandwidth W, and the squarer is a nonlinear device with input-output relationship

$$y = x^2.$$

To simulate this system with a digital computer, we should convert this continuous-time model into an equivalent discrete-time model as follows:



where S_i is the sample of $s(t)$ at the sampling epoch $t = iT_s$ with T_s the sampling interval and the finite impulse response (FIR) filter is an approximated counterpart of the low-pass filter in discrete-time. The white noise $n(t)$ cannot be simulated with its samples since each sample is a variable with infinite variance. Instead, a sequence $\{N_i\}$ of independent identically distributed Gaussian random variables with zero mean and variance σ^2 is used to be an equivalent counterpart of $n(t)$ in discrete-time.

(a) What is an appropriate sampling interval T_s? (10%)

(b) What is an appropriate variance σ^2 of N_i's to be used? (5%)

(Note that the answers should include reasonable explanations.)

4. A DSB-SC signal can be demodulated by an envelope detector if a sufficient amount of carrier is reinserted at the receiver. Suppose a carrier $A\cos(2\pi f_c t+\delta)$ is added to the received DSB-SC signal $m(t)\cos(2\pi f_c t)$, where $m(t)$ is the message, and the resulting signal is envelope detected. Assume $|m(t)| \ll A$.

   (a) Find the output after dc blocking. (5%)

   (b) Compare the effect of the phase error in this method with that in synchronous product demodulation. (5%)

5. It is desired to generate a stationary random signal $x(t)$ with the autocorrelation function

$$R_x(\tau) = e^{-|\tau|},$$

hence the power spectral density

$$S_x(f) = \frac{2}{1+(2\pi f)^2}.$$

We propose doing this in two ways:

I. By taking

$$x(t) = \int_0^\infty h(\alpha)n(t-\alpha)\,d\alpha,$$

where $n(t)$ is white noise of power spectral density $N_0/2$ with $N_0 = 1$ and $h(t)$ is some appropriate impulse response.

II. By setting $x(t) = \sqrt{2}\cos(2\pi f_0 t+\theta)$, where f_0 and θ are statistically independent random variables and θ is uniformly distributed in $(0, 2\pi)$.

   (a) Pick $h(t)$ to yield the desired power spectral density via method I. (4%)

   (b) Specify the probability density function for f_0 that yields the desired power spectral density in method II. (3%)

   (c) Sketch a typical sample function generated by method I; by method II. Do you expect them to look similar? Explain. (3%)

6. (a) The most common binary digital carrier modulations are OOK (on off keying), FSK (frequency shift keying), and PSK (phase shift keying). Based on the concept of signal space, quantitatively compare the performance of the three schemes, assuming coherent demodulation. (15%)

   (b) PCM (pulse code modulation) is also very common in digital communication systems. What is the difference among those digital modulations and PCM? (5%)

   (c) Suppose the quantization error for a PCM system must be less than $\pm$ 5%. Please find the minimum number of bits for each sample in this PCM scheme. (5%)