類組:電機類 科目:工程數學 C(3005)

共_12/頁第____頁

※請在答案卡內作答

- 本測驗試題為多選題(答案可能有一個或多個),請選出所有正確或最適當的答案,並請用2B鉛筆作答於答案卡。
- 共二十題,每題五分。每題ABCDE每一選項單獨計分;每一選項的個別分數為一分,答 錯倒扣一分。

Notation: In the following questions, underlined letters such as $\underline{a}, \underline{b}$, etc. denote column vectors of proper length; boldface letters such as A, B, etc. denote matrices of proper size; A^{\top} means the transpose of matrix A. I_n is the $(n \times n)$ identity matrix. $\|\underline{a}\|$ means the Euclidean norm of vector \underline{a} . \mathbb{R} is the usual set of all real numbers; \mathbb{C} is the usual set of all complex numbers. By $A \in \mathbb{R}^{m \times n}$ we mean A is an $m \times n$ real-valued matrix. u(x) is unit-step function defined as u(x) = 1 if $x \geq 0$ and u(x) = 0 if x < 0; \star is the convolution operator; $\mathcal{L}: f(x) \mapsto F(s)$ and $\mathcal{L}^{-1}: F(s) \mapsto f(x)$ denote the <u>unilateral</u> Laplace and inverse Laplace transforms for $x \geq 0$, respectively.

- \cdot Let $\mathbf{H} = \mathbf{I}_n 2\underline{u}\,\underline{u}^{\top}$, where $\underline{u} \in \mathbb{R}^n$, $n \geq 2$ and $||\underline{u}|| = 1$. Which of the following statements is/are true?
 - (A) The matrix **H** is both symmetric and orthogonal.
 - (B) Both 1 and -1 are eigenvalues of \mathbf{H} .
 - (C) det(H) = 1.
 - (D) Trace(H) = n-2.
 - (E) None of the above.

注:背面有試題

多子

類組: <u>電機類</u> 科目: 工程數學 C(3005)

共12頁第2頁

※請在答案卡內作答

- Two square matrices A and B are similar, denoted by $A \sim B$, if $B = P^{-1}AP$ for some nonsingular matrix P. Which of the following statements is/are true?
 - (A) Two similar matrices always have the same set of eigenvalues, including multiplicity.
 - (B) Two $n \times n$ matrices having the same set of eigenvalues, including multiplicity, are similar.
 - (C) Any two square matrices with the same trace and determinant are similar.
 - (D) If $A \sim B$, then $p(A) \sim p(B)$ for any polynomial p(x).
 - (E) None of the above.

- \exists Let $\mathbf{A} = \underline{x}\,\underline{y}^{\mathsf{T}}$, where \underline{x} and \underline{y} are two nonzero vectors of \mathbb{R}^n , n > 1. Which of the following statements is/are true?
 - (A) rank(A) = 1 and the range space of A is $Span\{y\}$.
 - (B) nullity(A) = 2 and the null space of A is $Span\{\underline{x}, y\}$.
 - (C) Trace(A) = 1 and det(A) = 0
 - (D) A is always diagonalizable.
 - (E) None of the above.

注:背面有試題



類組: <u>電機類</u> 科目: 工程數學 C(3005)

共 1 2 頁第 一页 頁

※請在答案卡內作答

Let $\mathbf{A} = \underline{x} \underline{y}^{\mathsf{T}} + \underline{y} \underline{x}^{\mathsf{T}}$, where \underline{x} and \underline{y} are two nonzero orthonormal vectors of \mathbb{R}^n and n > 2. Which of the following statements is/are true?

- (A) Both \underline{x} and y are eigenvectors of \mathbf{A} .
- (B) Trace(\mathbf{A}) = 1 and det(\mathbf{A}) = 0.
- (C) A is not diagonalizable.
- (D) The least square solution of $\mathbf{A}\underline{z} = \underline{b}$, where \underline{b} is a vector in \mathbb{R}^n , is $(\underline{b}^{\mathsf{T}}\underline{x})\underline{x} + (\underline{b}^{\mathsf{T}}y)y$.
- (E) None of the above.

 \pounds Let $A, B \in \mathbb{R}^{n \times n}$, and $\{\underline{u}_1, \dots, \underline{u}_n\}$ be an orthonormal basis for \mathbb{R}^n . It is known that $\langle A, B \rangle = \text{Trace}(A^{\top}B)$ is an inner product. We denote $A \perp B$ if $\langle A, B \rangle = 0$. Which of the following statements is/are true?

- (A) Let $\underline{x}, \underline{y}, \underline{w}, \underline{z}$ be four nonzero vectors of \mathbb{R}^n . Then $\underline{w} \underline{z}^{\top} \perp \underline{x} \underline{y}^{\top}$ if and only if $\underline{w} \perp \underline{x}$ and $\underline{z} \perp y$.
- (B) The set $\mathcal{B}_1 := \left\{ \underline{u}_i \, \underline{u}_j^\top : i, j = 1, \dots, n \right\}$ is an orthonormal basis for $\mathbb{R}^{n \times n}$.
- (C) The set

$$\mathcal{B}_{2} = \left\{ \underline{u}_{i} \, \underline{u}_{i}^{\mathsf{T}} + \frac{\underline{u}_{i} \, \underline{u}_{j}^{\mathsf{T}} + \underline{u}_{j} \, \underline{u}_{i}^{\mathsf{T}}}{\sqrt{2}} : 1 \leq i < j \leq n \right\}$$

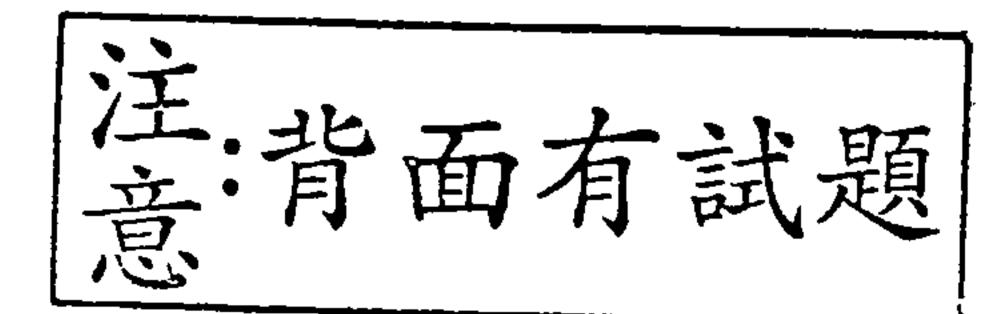
is an orthonormal basis for the real vector space $S_1 = \{ \mathbf{A} \in \mathbb{R}^{n \times n} : \mathbf{A} = \mathbf{A}^{\top} \}$.

(D) The set

$$\mathcal{B}_3 = \left\{ \frac{\underline{u}_i \, \underline{u}_j^{\mathsf{T}} - \underline{u}_j \, \underline{u}_i^{\mathsf{T}}}{\sqrt{2}} : 1 \le i < j \le n \right\}$$

is an orthonormal basis for the real vector space $\mathcal{S}_2 = \left\{ \mathbf{A} \in \mathbb{R}^{n \times n} : \mathbf{A} = -\mathbf{A}^{\top} \right\}$

(E) None of the above.



共12 頁第止頁

※請在答案卡內作答

 \Rightarrow The system of linear equations $\mathbf{A}\underline{x} = \underline{b}$ has

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

Which of the following statements is/are true?

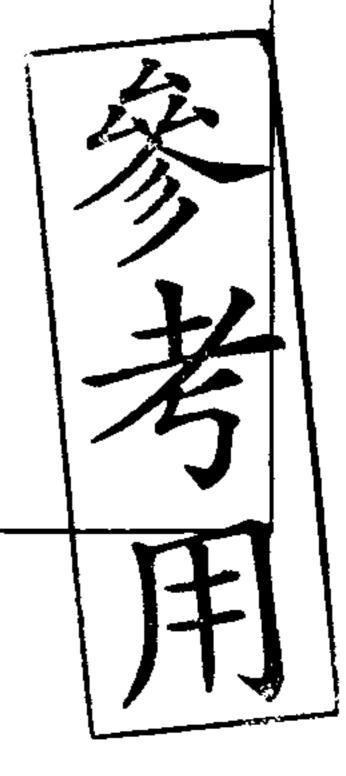
- (A) $rank(\mathbf{A}) + nullity(\mathbf{A}) = 3$.
- (B) $\mathbf{A}^{\mathsf{T}}\mathbf{A}$ is a symmetric 2×2 matrix.
- (C) The nullspace of A has two linearly independent vectors.
- (D) $\mathbf{A}^{\mathsf{T}}\mathbf{A}$ is an invertible matrix.
- (E) None of the above.

- 七、 Continued from Problem 六, which of the following statements is/are true?
 - (A) The matrix A has the same row space as the following matrix

$$\left[\begin{array}{cccc} 1 & 1 & 1 \\ 0 & 1 & 2 \end{array}\right]$$

- (B) $\det(\mathbf{A}\mathbf{A}^{\mathsf{T}}) = 0$.
- (C) Let \underline{p} be the projected vector of \underline{b} onto the column space of \mathbf{A} . The Euclidean distance between \underline{b} and \underline{p} is zero.
- (D) There exists a (3×2) matrix C such that rank(CA) = 3.
- (E) None of the above.

注:背面有試題



類組: <u>電機類</u> 科目: 工程數學 C(3005)

共12頁第5頁

※請在答案卡內作答

 \wedge Continued from Problem $\dot{\Rightarrow}$, let B_1 be a (2×2) matrix and consider the system $B_1A\underline{x} = B_1\underline{b}$. It is known that

$$\mathbf{B}_1 \underline{b} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \text{ and } \underline{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Which of the following vectors can be column vectors of \mathbf{B}_1 ?

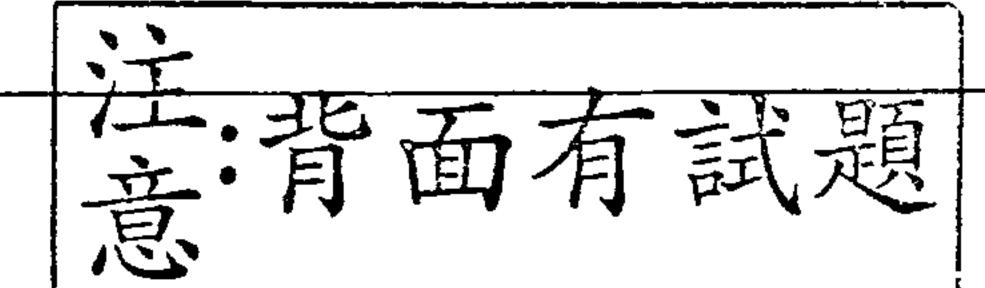
- $(A) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- (B) \begin{bmatrix} 0 \\ 0 \end{bmatrix}
- $(C) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- $(D) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- (E) None of the above.

九、 Given the matrices

$$\mathbf{M}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{M}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{M}_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

which of the following statements is/are true?

- (A) $\mathbf{M_1},\,\mathbf{M_2}$ and $\mathbf{M_3}$ are linearly independent over \mathbb{R} in $\mathbb{R}^{2\times 2}.$
- (B) The span of $\{M_1,M_2,M_3\}$ is the set of all (2×2) real matrices.
- (C) The set of all Hermitian (2×2) complex-valued matrices is a subspace of the span of $\{M_1, M_2, M_3\}$ over \mathbb{C} .
- (D) Any linear combination of $M_1,\,M_2$ and M_3 can be diagonalized over $\mathbb C.$
- (E) None of the above.





共 7 頁第 万 頁

※請在答案卡內作答

十、 Continued from Problem 九, let $B=3M_1+4M_2+M_3$ and $C=B^4-4B^3-9B^2+27B+11I_2$. Which of the following is/are true?

$$(A) \mathbf{C} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(B)
$$C = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

$$(C) \quad C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(D)
$$C = \begin{bmatrix} 3 & 1 \\ & & 1 \\ 1 & 4 \end{bmatrix}$$

(E) None of the above.

 $+-\cdot$ Solve the first-order differential equation $x^2y'(x)+xy(x)\ln(y(x))=xy(x)$. Which of the following statements is/are true?

- (A) This is a homogeneous and linear differential equation.
- (B) y(x) = 0 is one particular solution.
- (C) $y(x) = \exp(1)$ is another particular solution.
- (D) x = 0 is also a solution.
- (E) None of the above.

注:背面有試題

類組: <u>電機類</u> 科目: 工程數學 C(3005)

共一一頁第一頁

※請在答案卡內作答

+ : Continued from Problem + . Given the initial condition y(a) = b, which of the following statements is/are true?

- (A) No solution if a = 0.
- (B) A unique solution if b = a > 0.
- (C) More than one solution if $b = \exp(1)$.
- (D) A unique solution if $a \neq 0$ and b > 0.
- (E) None of the above.

+ \equiv . The second-order linear differential equation $(1-x^2)y''(x)+2xy'(x)-2y(x)=f(x)$, for -1 < x < 1. To find the homogeneous solution, i.e. f(x)=0, given one solution $y_1(x)=x$, the other linearly independent solution $y_2(x)$ can be derived by setting $y_2(x)=v(x)y_1(x)$. Assuming v(x) satisfies v(1)=2 and $v(2)=\frac{5}{2}$, which of the following statements about v(x) is/are true?

(A)
$$x(1-x^2)v''(x) + 2v'(x) = 0$$
.

(B)
$$x(x^2 - 1)v''(x) + 2v'(x) = 0$$
.

(C)
$$v'(x) = \frac{x^2-1}{x^2}$$
.

(D)
$$v'(x) = \frac{x^2}{x^2 - 1}$$
.

這背面有試題

多手

共 工 頁 第 8 頁

※請在答案卡內作答

十四、 Continued from Problem 十三, find a particular solution for $f(x)=1-x^2$ by the method of variation of parameters, i.e., $y_p(x)=u_1(x)y_1(x)+u_2(x)y_2(x)$, where $y_1(x)$ and $y_2(x)$ are obtained from Problem 十三. Which of the following statements regarding $u_1(x)$ and $u_2(x)$ can be true?

(A)
$$u_1(x) = \ln(1+x) - \ln(1-x) - x$$
.

(B)
$$u_2(x) = \ln(1+x) + \ln(1-x)$$
.

(C)
$$u_1(x) = x + \frac{x^3}{3}$$
.

(D)
$$u_2(x) = -\frac{x^2}{2}$$
.

(E) None of the above.

+£. Solve the initial value problem of $(2x-x^2)y''(x)-5(x-1)y'(x)-3y(x)=0$ with y(1)=0 and y'(1)=1 by power series of the form $y(x)=\sum_{n=0}^{\infty}c_n(x-1)^n$. Which of the following statements regarding the recurrence relation as well as values of coefficients c_n is/are true?

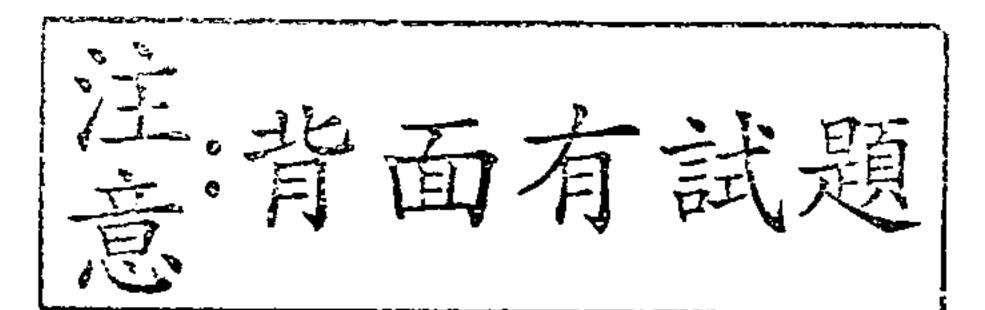
(A)
$$c_{n+2} = \frac{n+2}{n+3}c_n$$
.

(B)
$$c_8 = 0$$
.

(C)
$$c_5 = \frac{8}{5}$$
.

(D)
$$c_9 = \frac{63}{128}$$
.

(E) None of the above.







類組: <u>電機類</u> 科目: 工程數學 C(3005)

共12頁第一頁

※請在答案卡內作答

+ \div Let $\underline{y}(x) = [y_1(x) \ y_2(x)]^{\mathsf{T}}$ and consider the following system of first-order differential equations

$$\underline{y}'(x) = \begin{bmatrix} 6 & -7 \\ 1 & -2 \end{bmatrix} \underline{y}(x) + \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Assuming $\underline{y}(0) = [1 \ -1]^{\top}$, let $\underline{Y}(s) = [Y_1(s) \ Y_2(s)]^{\top} = \mathcal{L}\{\underline{y}(x)\}$. Which of the following statements is/are true?

- (A) $Y_1(1) = \frac{5}{8}$.
- (B) $Y_1(6) = \frac{85}{42}$.
- (C) $Y_1(7) Y_2(7) = \frac{11}{14}$.
- (D) $\frac{Y_2(8)}{Y_1(8)} = 0$.
- (E) None of the above.

++ · Continued from Problem + \dot{x} , which of the following statements regarding the solution $\underline{y}(x)$ is/are true?

- (A) $y_2'(0) = 6$.
- (B) $y_1''(0) = 8$.
- (C) $y_2''(0) = 3$.
- (D) $\lim_{x\to -\infty} (y_1(x) y_2(x)) = \frac{1}{4}$.
- (E) None of the above.

涟:背面有試題



類組: <u>電機類</u> 科目: 工程數學 C(3005)

共 () 頁第 () 頁

※請在答案卡內作答

十八、 Let y(x) be a real-valued function satisfying the following second-order differential equation

$$y''(x) + y(x) = f(x)$$

Assume y(0) = y'(0) = 0 and $f(x) = \sum_{n\geq 0} u(x - n\pi) \sin(x - n\pi)$. Which of the following statements is/are true?

- (A) y(x) is a periodic function with period π .
- (B) y(x) has a Fourier-series representation for all x > 0.
- $(C) y\left(\frac{\pi}{2}\right) = \frac{1}{2}.$
- (D) $y'\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$.
- (E) None of the above.

意普面有試題

共 2 頁第1 頁

※請在答案卡內作答

十九、 Let $f(x) = \frac{\pi - x}{2}$ and

$$g(x) = [f(x) (u(x) - u(x - \pi))] \star \left(\sum_{n = -\infty}^{\infty} \delta(x - n2\pi)\right)$$

It is known that g(x) has the following Fourier series representation

$$\tilde{g}(x) = \sum_{n=0}^{\infty} \left(a_n \cos \left(\frac{n2\pi}{T} x \right) + b_n \sin \left(\frac{n2\pi}{T} x \right) \right).$$

with minimal period T and Fourier series coefficients a_n and b_n . Which of the following statements is/are true?

- (A) $a_4 = 0$.
- (B) $b_4 = \frac{1}{8}$.
- (C) $\tilde{g}(2\pi) = \frac{\pi}{4}$.
- (D) $\sum_{n=1}^{\infty} a_n^2 + b_n^2 = \frac{5}{96}\pi^2$.
- (E) None of the above.

類組: <u>電機類</u> 科目: 工程數學 C(3005)

共一頁第一一頁

※請在答案卡內作答

二十、 Continued from Problem 十九. Consider the following boundary value problem for the bivariate function y(x,t)

$$\frac{\partial}{\partial t}y(x,t) = \frac{\partial^2}{\partial x^2}y(x,t),$$

for $x \in (0,\pi)$ with initial conditions $y(0,t) = y(\pi,t) = 0$ and y(x,0) = xf(x), where f(x) is given in Problem $+\pi$. The solution y(x,t) can be representation in the following form

$$y(x,t) = \sum_{n\geq 0} c_n e^{-d_n t} \sin(e_n x)$$

for some $c_n, d_n, e_n \in \mathbb{R}$. Which of the following statements is/are true?

- (A) $c_1 = \frac{4}{\pi}$.
- (B) $c_2 = 1$.
- (C) $d_3 = 3$.
- (D) $e_3 = 3$.
- (E) None of the above.

涟.背面有試題