類組: 電機類 科目: 工程數學 A(3003)

共 3 第 4 頁

※請在答案卷內作答

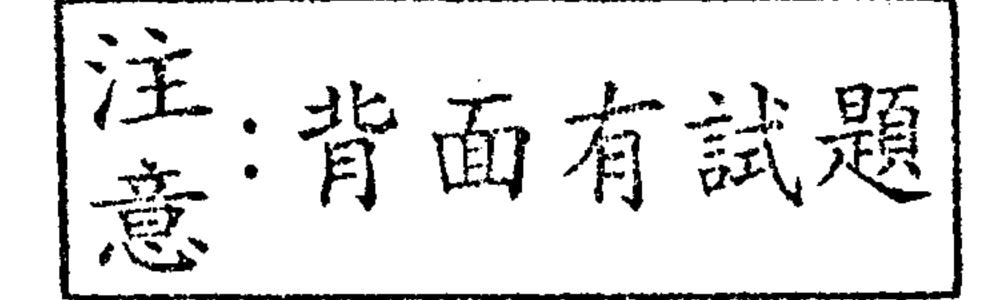
Note: Detailed derivations are required to obtain a full score for each problem.

1. (10%) Let
$$A = \begin{pmatrix} 0 & 3 & 2 & 1 & -4 \\ 2 & 10 & 10 & 16 & 14 \\ -3 & 0 & -5 & -2 & -7 \\ -2 & -1 & -4 & -3 & -6 \\ 2 & 7 & 8 & 11 & 10 \end{pmatrix}$$
 and $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$.

- (a) (3%) Compute rank(A).
- (b) (2%) Compute rank(AB).
- (c) (3%) Compute $rank(A^tAAA^t)$.
- (d) (2%) Compute $\dim(N(B^tA))$.
- 2. (10%) Let V be the vector space spanned by the ordered basis functions $\beta = \{xe^{ax}, e^{ax}, e^{bx}\}$ where $a, b \in R$ and $a \neq b$. Define a linear transformation $T : V \to V$ with parameters $p, q \in R$:

$$T(y(x)) = y'' + py' + qy.$$

- (a) (4%) Find the matrix representation for $[T]_{\beta}$.
- (b) (6%) There are two conditions for p and q such that $\dim(N(T)) = 2$. For each condition, express p and q in terms of a and b, and also find the corresponding null space.
- 3. (5%) Let A and B are $n \times n$ square matrices such that AB = C where C is an upper triangular matrix with $C_{ij} \neq 0$ whenever $j \geq i$. Prove that A and B are both invertible.
- 4. (16%) Let V be a vector space over a field \mathbb{F} , T be a linear operator on V, and W be a subspace of V. We say that W is invariant under T if for each vector v in W the vector ∇v is also in W. Let W be an invariant subspace for T, and $v \in V$. The T-conductor of v into W, denoted by $S_{\mathsf{T}}(v,\mathsf{W})$, is defined as the set of all polynomials g(x) over \mathbb{F} such that $g(\mathsf{T})v$ is in W, i.e., $S_{\mathsf{T}}(v,\mathsf{W}) = \{g(x) \in \mathbb{F}[x] | g(\mathsf{T})v \in \mathsf{W}\}$.
 - (a) (8%) Prove the following statement. If W is an invariant subspace for T, then, for each polynomial $g(x) \in \mathbb{F}[x]$, W is invariant under g(T).
 - (b) (8%) Prove that if W is an invariant subspace for T then $S_T(v, W)$ is a subspace of $\mathbb{F}[x]$, the set of polynomials over \mathbb{F} .
- 5. (9%) Let T be a linear operator on a finite-dimensional inner product space V. Prove that $N(T^*T) = N(T)$, where N(T) is the null space for T.



台灣聯合大學系統105學年度碩士班招生考試試題

類組:<u>電機類</u>科目:工程數學A(3003)

共_3 第 2 頁

※請在答案卷內作答

6. (5%) Solve
$$y'' + 5y' + 4y = 10e^{-3x}$$

7. (5%) Solve the following differential equation with the initial conditions by using Laplace transform. u(t) is the unit step function.

$$y'' - 2y' + y = (e^t + t)u(t)$$

 $y(0) = 1$
 $y'(0) = 0$

8. (5%) Find the eigenvalues and eigenfunctions.

$$y'' + \lambda y = 0$$
; $y(0) = y(\pi/2) = 0$

- 9. (5%) Find the general solution of $(1-x^2)y'' 2xy' + 12y = 0$ using "series solution" when -1 < x < 1.
- 10. (5%) Find the Fourier Series of f(t).

$$f(t) = \begin{cases} 0, -\pi < t < 0 \\ 1, 0 < t < \pi \end{cases}$$
$$f(t + n2\pi) = f(t)$$

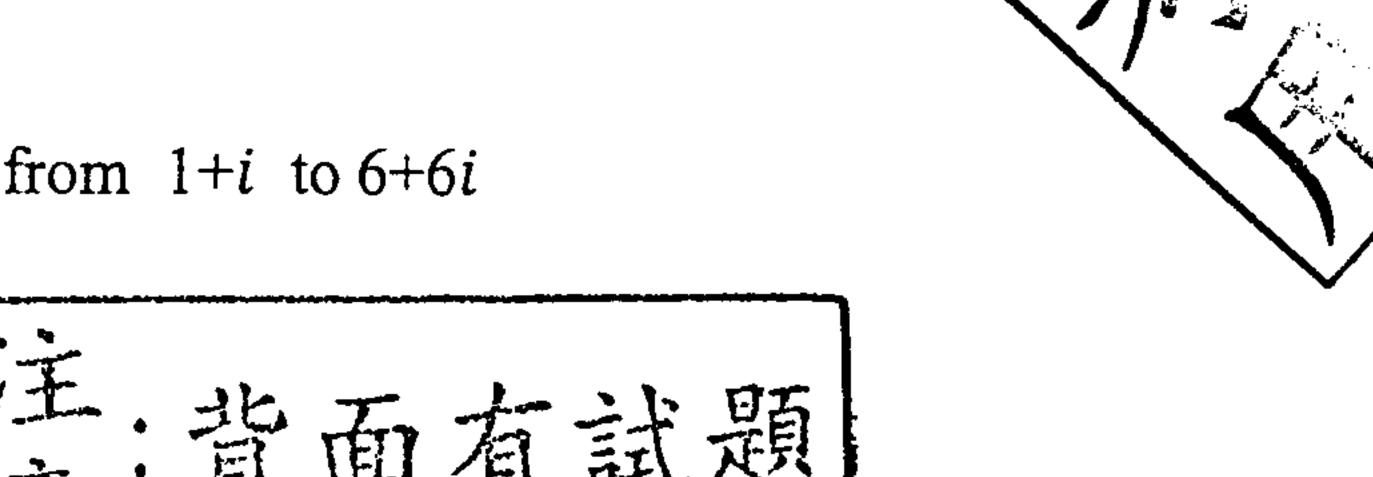
- 11. (5%) Find the principle value of $(3 + 4i)^{1/3}$.
- 12. (5%) Find the open disk of convergence of the following power series and its radius

$$\sum_{n=0}^{\infty} \frac{n^3}{4^n} (z + 3i)^{3n}$$

13. (5%) Evaluate the integration of

$$\int_{C} \mathcal{R}e(z)dz$$

where C is the shortest path from 1+i to 6+6i



台灣聯合大學系統105學年度碩士班招生考試試題

類組: <u>電機類</u> 科目: <u>工程數學 A(3003)</u>

共 3 第 3 頁

※請在答案卷內作答

14. (10%) Evaluate the integration of

$$\oint_C \frac{\sinh z}{\sin z} dz$$

where C: $|z| = \frac{4}{3}\pi$, clockwise.

