類組：電機類　科目：離散數學(300C)　※請在答案卷內作答

一.　填空題(共 20 題，每題 4 分：合計 80 分)

答題說明：
1. 請**依題號順序**書寫於答案卷，並清楚標註題號 。
2. 題號 1-10 題目詢問內容描述正確與否(題目前標註(T or F)者)，認為描述正確者書寫 T，錯誤者書寫 F。其餘答案一律不給分。
3. 其餘題目(11-20)請直接書寫答案，無需計算過程。

1. (**T** or **F**) Relation $\boldsymbol{R} = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$ is an equivalence relation on the set $A = \{1,2,3\}$.

2. (**T** or **F**) Let $\boldsymbol{P} = \{a, b, c\}$ and $\boldsymbol{Q} = \{x,\ y,\ z\}$. $\otimes$ and $\oplus$ are two operators defined upon ***P*** and ***G***, respectively, as follows

$\otimes$	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

$\oplus$	x	y	z
x	x	y	z
y	y	x	z
z	z	x	y

Then, $\{\boldsymbol{P},\otimes\}$ and $\{\boldsymbol{Q},\oplus\}$ are homeomorphic.

3. (**T** or **F**) Let $X = \{0,1,2,\dots\}$ and $Y = \{-1,-2,-3,\dots\}$. Then $Z = X \cup Y$ is *countably infinite*.

4. (**T** or **F**) For a set, the $\oplus$ operation is neither left nor right distributive with respect to the $\cap$ operation.

5. (**T** or **F**) Let $\boldsymbol{R}$ be a precedence relation on the set $\boldsymbol{Z}^+$ and $(x,y) \in \boldsymbol{R}$ iff $x|y\ \forall x,y \in \boldsymbol{Z}^+$. Then $(\boldsymbol{Z}^+, |)$ is a partial order set (POS).

注意:背面有試題

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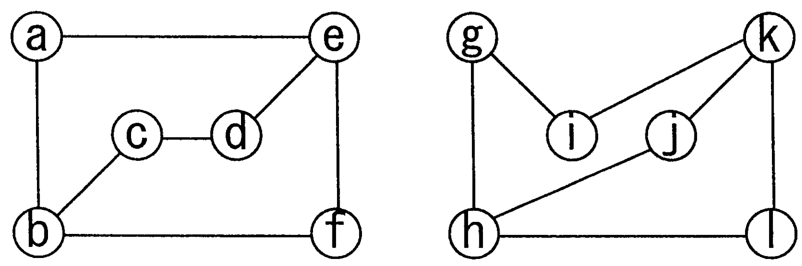
6. (**T** or **F**) The following two graphs are isomorphic.



7. (**T** or **F**) If a graph $\boldsymbol{G} = \{\boldsymbol{V},\ \boldsymbol{E}\}$ is a simple connected graph, then $|\boldsymbol{E}| \leq \frac{|V|(|V|-1)}{2}$.

8. (**T** or **F**) Suppose that the relation ***R*** on a set is represented by

$$\boldsymbol{M}_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Then ***R*** is both reflexive and symmetric

9. (**T** or **F**) Determine if the conclusion follows from the given premises.
   P1: $\forall x\ \exists y\ [P(x,\mathrm{y}) \wedge S(x,y)]$
   P2: $\forall x\ \forall y\ [P(x,\mathrm{y}) \Rightarrow R(x,y)]$
   C: $\forall x\ \exists y\ [R(x,\mathrm{y}) \wedge S(x,y)]$

10. (**T** or **F**)
    $(\overline{\mathrm{w}} + \overline{\mathrm{y}} + \overline{\mathrm{z}})\ (\mathrm{x} + \overline{\mathrm{y}} + \overline{\mathrm{z}})\ (\mathrm{w} + \overline{\mathrm{x}} + \mathrm{z})\ (\mathrm{w} + \mathrm{y} + \mathrm{z})\ (\overline{\mathrm{w}} + \mathrm{x} + \overline{\mathrm{y}})$ is unstaisfiable.

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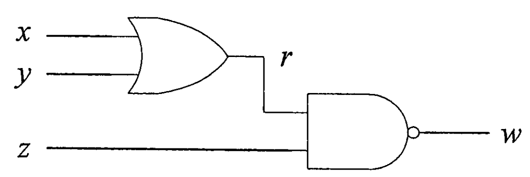
11. Please derive the **conjunctive normal form (CNF)** for the following circuit:

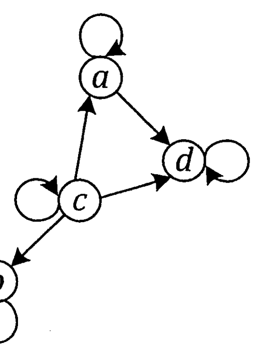


Note that the negated of a Boolean variable x should be represented as $\bar{x}$.

12. Solve the following recurrence:
$a_n = 5a_{n-1} - 6a_{n-2} + n$ where $a_0 = 2$ and $a_1 = 1$
(Hint: use the characteristic roots method)

13. Derive the number of edges for a forest with *n* vertices and *m* components.

14. Given a partially ordered graph as follows,



Please draw the corresponding Hasse diagram.

15. Let *G* be the grammar with vocabulary $V = \{S, A, a, b\}$, set of terminals $T = \{a, b\}$, starting symbol *S* and productions $P = \{S \rightarrow aA, S \rightarrow b, A \rightarrow aa\}$. What is *L*(*G*) the language of this grammar?

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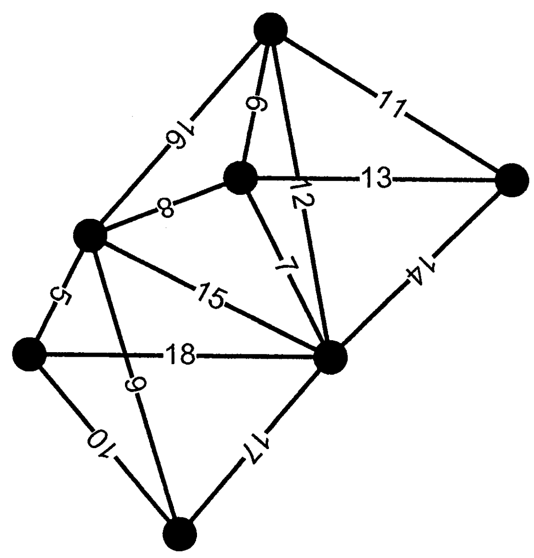
16. How many solutions are there , in nonnegative integers, to the following equation:

$$x_1 + x_2 + x_3 + x_4 = 16$$

17. Determine a minimum spanning tree for the following graph.



18. Draw a binary tree to represent $(a - b) \div c + d \times (e - f \div g)$

19. Let $\boldsymbol{R} = \{(x, y): y \leq x^2 - 5\}$ and $\boldsymbol{S} = \{(x, y): y = x^2 + 2x + 3\}$
What is the composition of relation $\boldsymbol{R} \circ \boldsymbol{S}$?

20. Derive a general formula for the recurrences of the form

$$\boldsymbol{T}(n) = a\boldsymbol{T}\left(\frac{n}{b}\right) + n^c$$

where a, b and c are constants, n is a power of b and $\boldsymbol{T}(1) = k$.

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二. 問答/計算題(共4大題，每題5分：合計20分)

答題說明：1. 請**依題號順序**書寫於答案卷，並清楚標註題號。

2. 每題題目前說明配分。例如：(5分)即代表本題中此題五分。以此類推。

21. (5分) Let A, B and C define as follows,

A: $\forall x\,(F(x) \Rightarrow G(x))$

B: $\exists x\,(F(x) \wedge \forall y(G(y) \Rightarrow \neg R(x,y)))$

C: $\exists x\,(G(x) \wedge \forall y(F(y) \Rightarrow \neg R(x,y)))$,

Please prove that C is a consequence of A and B.

22. (5分)

Let $F(n)$ be expressed by the Cantor expansion as

$$F(n) = a_n n! + a_{n-1}(n-1)! + \cdots + a_1 1! = \sum_{i=1}^{n} a_i i!$$

where a_i is an integer for i=1,..,n.

Use the mathematical induction to prove that

$$f(n) < (n+1)! \text{ for } n > 0 \text{ and } f(n) \in F(n)$$

23. (5分)

For a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group.

24. (5分)

Given $L_1 = \{a^j b^j | i,j \geq 1\}$

and $L_2 = \{a^j b^j | i,j \geq 0 \text{ and } i+j \text{ is even}\}$,

let $\bar{L}$ denote the complement of L over $\{a, b\}$ where L is a subset of $\{a, b\}^*$. Construct a grammar for the language $L_1 \cap L_2$.