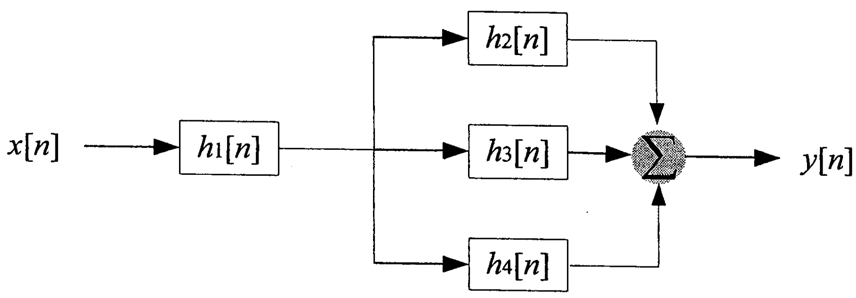
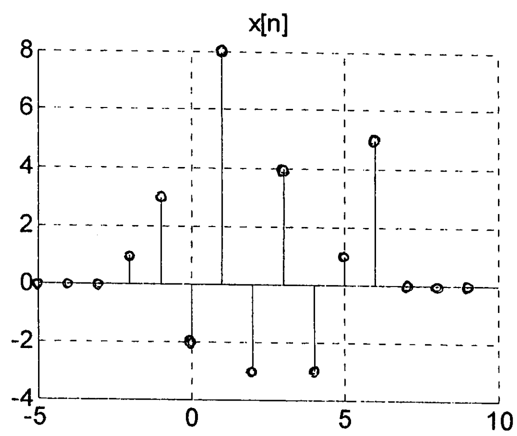
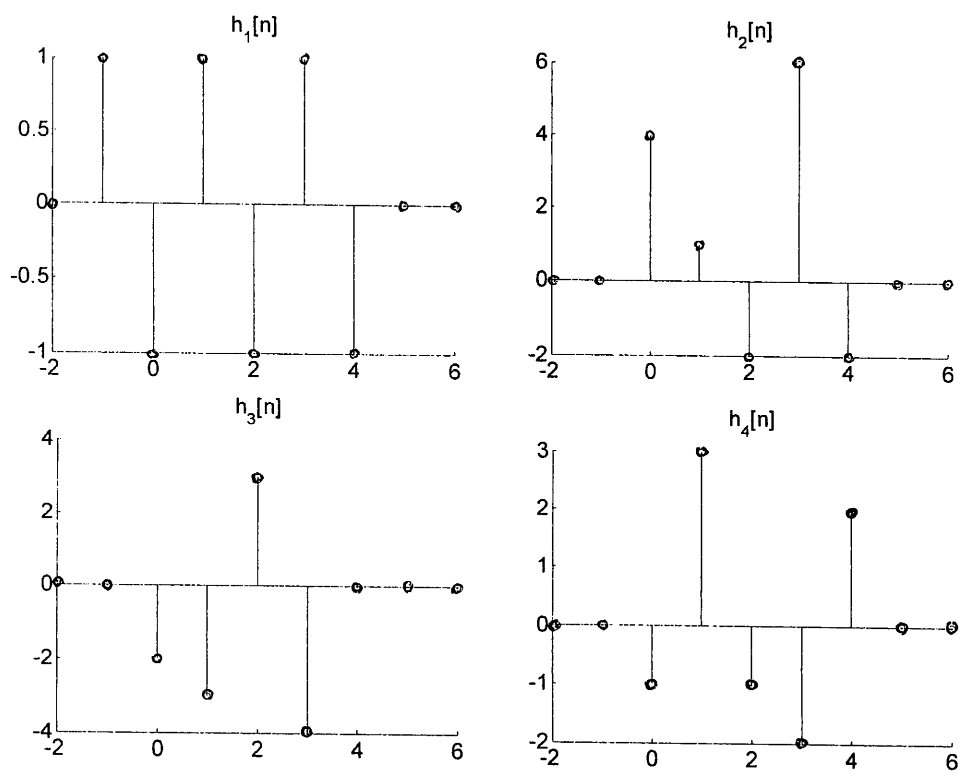
類組：電機類　科目：訊號與系統(300B)　　※請在答案卷內作答

一、(10 points) Given the following sub-system impulse response and input $x[n]$. Obtain the output $y[n]$.



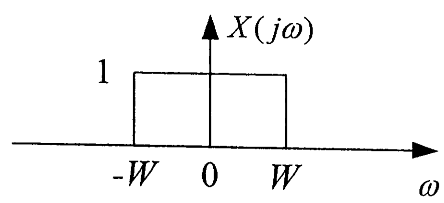




注意：背面有試題

類組：電機類　科目：訊號與系統(300B)　　　　※請在答案卷內作答

二、(15 points) The frequency spectrum of a signal $x(t)$ is shown below. Answer the following questions:




(一)、(5 points) Let the **sampling frequency** $\omega_s = 4W$. Plot the frequency spectrum of $x_\delta(t)$, where $x_\delta(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_s)$.

(二) (5 points) Following (a), plot the frequency spectrum of $x[n]$.

(三) (5 points) Let the sampling frequency $\omega_s = \frac{3}{2}W$. Plot the frequency spectrum of $x[n]$.

三、(10 points) Find the **Laplace transform** of $x(t) = \frac{d^2}{dt^2}(e^{-3(t-2)}u(t-2))$.

四、(15 pts) Let $X(z) = \frac{z^{-1}}{1 - 2.5z^{-1} + z^{-2}}$ and $Y(z) = \frac{0.5z^{-1}}{1 - 2z^{-1} + 0.75z^{-2}}$. Answer the following questions:

(一) (3 pts) Find the **region of convergence** so that $X(z)$ is stable. Also, find the corresponding time domain sequence $x[n]$.

(二) (3 pts) Find the region of convergence so that $Y(z)$ is causal. Also, find the corresponding time domain sequence $y[n]$.

(三) (9 pts) Let $X(z)$ be the input and $Y(z)$ be the output. $X(z)$ has the same condition in (a) and $Y(z)$ has the same condition in (b), find the system transfer function $H(z)$. Is $H(z)$ stable? Is $H(z)$ casual? Explain your answer.

五、(5 pts) A continuous-time linear system S with input $x(t)$ and output $y(t)$ yields the following input-output pairs:

$$x(t) = e^{j2t} \xrightarrow{S} y(t) = e^{j3t}$$
$$x(t) = e^{-j2t} \xrightarrow{S} y(t) = e^{-j3t}$$

(一)(2 pts) Is this system linear time-invariant? Just simply answer yes or no.

(二)(3 pts) The corresponding output of the system when input = cos(2($t$-0.5)) is ________.

六、(15 pts) Find the **Fourier series representation** of the outputs for each of the following cases:

(一)(5 pts) The input signal $x(t) = \sin(4\pi t) + \cos(6\pi t + \pi/4)$ is inputted to a causal continuous-time linear time-invariant system whose input $x(t)$ and output $y(t)$ are related by $\frac{d}{dt}y(t) + 4y(t) = x(t)$.

(二)(5 pts) The input signal $x[n] = \sin(\frac{3}{4}\pi n)$ is inputted to a causal discrete-time linear time-invariant system whose input $x[n]$ and output $y[n]$ are related by $y[n] - \frac{1}{4}y[n] = x[n]$.

注意：背面有試題

類組：電機類　科目：訊號與系統(300B)　※請在答案卷內作答

(三)(5 pts) The input signal $x[n]=\sum_{k=-\infty}^{\infty}\delta[n-4k]$ is inputted to a discrete-time linear time-invariant system with impulse response $h[n]=\begin{cases}1, & 0\le n\le 2\\ -1, & -2\le n\le -1\\ 0, & otherwise\end{cases}$.

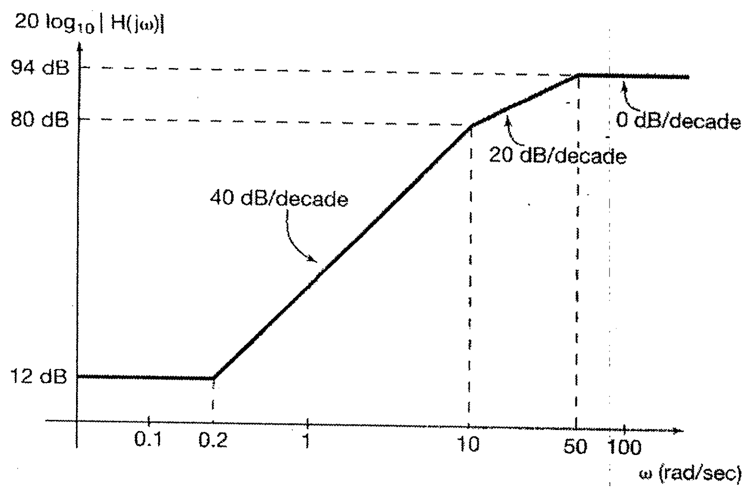
七、(15 pts) Let $p_T(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$ where $\delta(t)$ is the unit impulse (or Dirac delta) function, $T>0$ is a constant, and $x(t)=\operatorname{sin}c\left(\frac{t}{T}\right)\cos\left(\frac{\pi t}{2T}\right)$ with $\operatorname{sin}c(t)=\begin{cases}\sin(\pi t)/(\pi t), & t\ne 0\\ 1, & t=0\end{cases}$

(一) (6 pts) Find the **Fourier transform** (spectrum), $X(\omega)$, of $x(t)$ and plot it.

(二) (5 pts) Find the spectrum, $Xp(\omega)$, of $x_p(t)=x(t)p_T(t)$. Is it possible to recover $x(t)$ from $x_p(t)$? Why?

(三) (4 pts) If $p_T(t)$ is passed through a continuous-time linear time-invariant filter whose impulse response is $h(t)=x(t)$, find the **spectrum $Y(\omega)$** of the output $y(t)$ of the filter.

八、(5 pts) The straight-line approximation of the Bode magnitude plot of a causal and stable continuous-time LTI system S is shown below. Specify the **frequency response $H_i(j\omega)$** of a system that is the inverse of S.




九、(10 pts) Consider the following three frequency responses $H_1(e^{j\omega})$, $H_2(e^{j\omega})$, and $H_3(e^{j\omega})$ for causal and stable third-order linear time-invariant systems:

$$H_1(e^{j\omega})=\frac{1}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})},$$

$$H_2(e^{j\omega})=\frac{1}{(1+\frac{1}{2}e^{-j\omega})(1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})},$$

$$H_3(e^{j\omega})=\frac{1}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{3}{4}e^{-j\omega}+\frac{9}{16}e^{-j2\omega})}.$$

We want to determine which system(s) has(have) **oscillatory step-response(s)**. The following are 7 possible answers: (A) only $H_1(e^{j\omega})$, (B) only $H_2(e^{j\omega})$, (C) only $H_3(e^{j\omega})$, (D) only $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$, (E) only $H_1(e^{j\omega})$ and $H_3(e^{j\omega})$, (F) only $H_2(e^{j\omega})$ and $H_3(e^{j\omega})$, (G) all of them. Which one of the above 7 is the correct answer?