

類組：電機類 科目：離散數學(300C)

※請在答案卷內作答

## 一. 填空題(共 20 題，每題 4 分：合計 80 分)

- 答題說明：
1. 請依題號順序書寫於答案卷，並清楚標註題號。
  2. 題號 1-10 題目詢問內容描述正確與否(題目前標註(T or F)者)，認為描述正確者書寫 T，錯誤者書寫 F。其餘答案一律不給分。
  3. 其餘題目(11-20)請直接書寫答案，無需計算過程。

1. (T or F) Given a set  $\{P, Q, R\}$  of Boolean variables and the following well formed formulae (wff)

$$(P \vee Q \vee R) \wedge (\bar{P} \vee \bar{Q} \vee \bar{R}) \wedge (\bar{P} \vee Q) \wedge (\bar{Q} \vee R)$$

is *satisfiable*.

2. (T or F) The halting problem ( $HPS$ ) on input  $i$  is defined by

$$HPS_i = \begin{cases} 1, & \text{if there is at least one } x \in N \text{ for which } P_i(x) \text{ halts;} \\ 0, & \text{otherwise.} \end{cases}$$

Then,  $HPS_i$  is a *computable* function.

3. (T or F)  $2^{\{0,1\}^*}$  is *countably infinite*.
4. (T or F) For a set, the  $\cap$  operation is neither left nor right distributive with respect to the  $\times$  operation.
5. (T or F) The complete bipartite graph  $K_{3,3}$  is planar

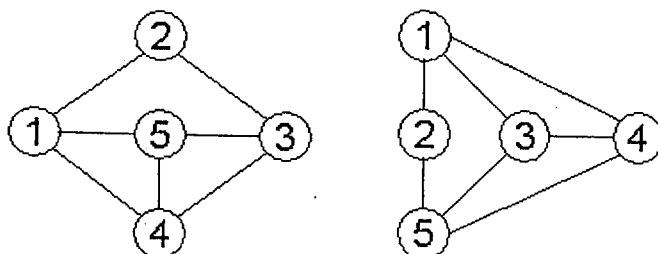
參考用

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6. (T or F) The following two graphs are isomorphic.



7. (T or F) If a graph  $G$  is an undirected graph, the number of vertices with odd degree is even.

8. (T or F) An additive group can be isomorphic to a multiplicative group.

9. (T or F) Let Ackermann's function  $A$  be defined as

$$A(x, y) = \begin{cases} y + 1, & \text{if } x = 0 \\ A(x - 1, 1), & \text{if } y = 0 \\ A(x - 1, A(x, y - 1)), & \text{otherwise} \end{cases}, \forall x, y \geq 0, x, y \in \mathbb{Z}$$

Ackermann's function  $A$  is a total function from  $\mathbb{N} \times \mathbb{N}$  to  $\mathbb{N}$ .

10. (T or F) If  $R_1$  and  $R_2$  are two equivalence relations, then  $R_1 \cup R_2$  is also an equivalence relation.

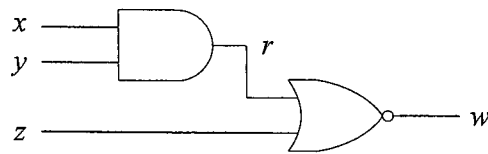
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11. Please derive the **conjunctive normal form (CNF)** for the following circuit:



Note that the negated of a Boolean variable  $x$  should be represented as  $\bar{x}$ .

12. Solve the following recurrence:

$$nf(n) = (n-1)f(n-1) + (n-2)f(n-2) + 2,$$

where  $\forall n \leq 2$  and  $f(0) = f(1) = 0$

(Hint: use the characteristic roots method)

13. Derive a formula for  $\sum_{i=1}^n i^3 \binom{n}{i}$  where  $n \in \mathbb{N}$ .

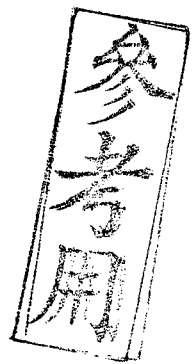
14. Given a partially ordered set  $P = \{2, 3, 5, 7, 11, 13, 17\}$  with the partial order  $\preceq$  as “ $m$  divides  $n$ ”, please draw the corresponding Hasse diagram.

15. Let  $A = B = C = \mathbb{R}$  where  $\mathbb{R}$  is the set of all real numbers. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  are defined respectively by

$$f(x) = 2x^2 + 3$$

$$g(x) = 5x - 1,$$

please derive  $f \circ g \circ f(x): \mathbb{R} \rightarrow \mathbb{R}$ .



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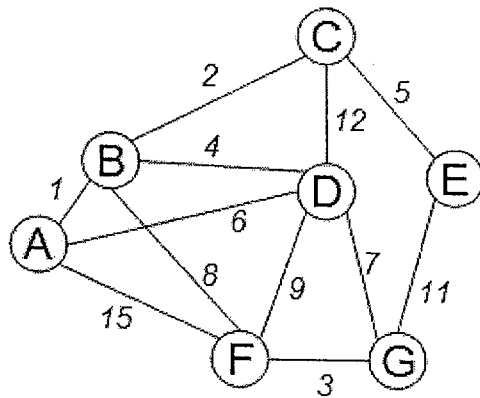
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16. How many solutions are there, in positive integers, to the following equation:

$$x_1 + x_2 + \cdots + x_{10} = 15$$

17. Derive the cost of the minimum spanning tree for the following graph



18. Solve the following recurrence relation and give a  $\Theta$ -bound for

$$T(n) = \sqrt{n}T(\sqrt{n}) + n$$

19. Solve the following recurrence relation and give a  $\Theta$ -bound for

$$T(n) = 49T\left(\frac{n}{25}\right) + n^{\frac{3}{2}} \lg n \text{ where } \lg n \equiv \log_2 n$$

20. Solve the following recurrence relation and give a  $\Theta$ -bound for

$$T(n) = \begin{cases} 8T\left(\frac{n}{2}\right) - n^3 + 7n^4 + 5, & n > 1 \\ 1, & n = 1 \end{cases}$$

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## 二. 問答/計算題(共 3 大題：合計 20 分)

- 答題說明： 1. 請依題號順序書寫於答案卷，並清楚標註題號。  
2. 每題題目前說明小題配分。例如：(5 分+5 分)即代表本題中第一小題五分與第二小題五分。以此類推。

21. (5 分)

Let  $\oplus$  denote the exclusive operator  $x \oplus y = (x \vee y) \wedge (\bar{x} \vee \bar{y})$ .  
Show that

$$\overline{\bigoplus_{1 \leq i \leq k} X_i} = \bar{X}_j \oplus X_1 \oplus \cdots \oplus X_{j-1} \oplus X_{j+1} \oplus \cdots \oplus X_k.$$

22. (5 分)

Please show the following two equations,

$$a_1 + a_2 + \cdots + a_5 = 11 \text{ and } b_1 + b_2 + \cdots + b_{12} = 4,$$

have the same number of solutions in nonnegative integers

23. (5 分+5 分)

Given a character set and the frequency of each character,

<i>character</i>	<i>frequency</i>
A	24
E	16
I	35
O	18
U	7

- (1) Find a Huffman code for the character set using the given frequencies. Show your derivation.  
(2) Decode the message 01000110001011

參考用