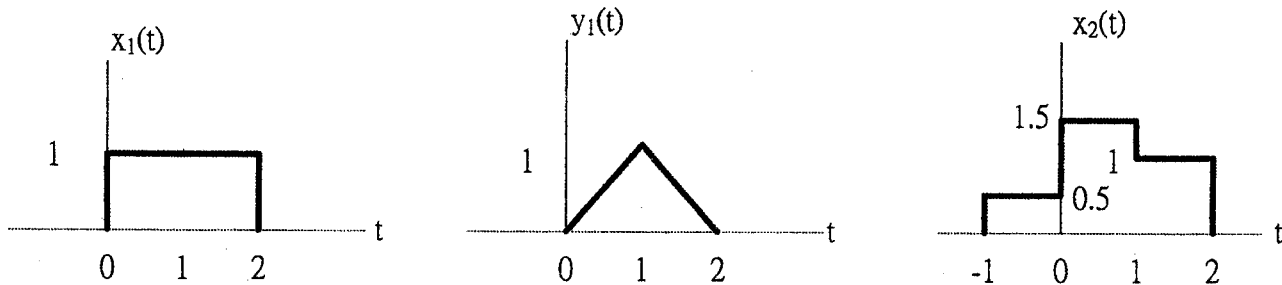類組：電機類　科目：訊號與系統（300B）　　　　　　　　　　　　<u>※請在答案卷內作答</u>

一、(5%) Consider a linear time-invariant (LTI) system whose response to $x_1(t)$ is the signal $y_1(t)$ as shown below. Determine the output of the LTI system (say, $y_2(t)$) when input is $x_2(t)$ shown below. You must

(一) (2%) first give an expression of $x_2(t)$ in terms of $x_1(t)$, and then

(二) (3%) sketch $y_2(t)$.

二、(10%) Consider a linear phase LTI system with frequency response $H(e^{j\omega})$ and group delay $\tau(\omega)$. Suppose $|H(e^{j\pi/2})| = 2$ and $\angle H(e^{j0}) = 0$, and $\tau(\pi/2) = 2$. Determine the answers of the following questions.

(一) The output of the system when input is $\cos(\pi n/2)$ = _____ (5%).

(二) The output of the system when input is $\sin(7\pi n/2 + \pi/4)$ = _____ (5%).

You need to write down your answers only. No partial scores for your computation procedures.

三、(10%) Consider a causal LTI system whose frequency response is given as: $H(e^{jw}) = e^{-jw} \dfrac{1 - \frac{1}{2}e^{jw}}{1 - \frac{1}{2}e^{-jw}}$.

Determine the answers of the following questions.

(一) $\left| H(e^{jw}) \right|$ = _____ (3%).

(二) The group delay $\tau(\omega)$ of this filter = _____ (3%).

(三) The output of this filter when the input is $\cos(\frac{\pi}{3}n)$ = _____ (4%)

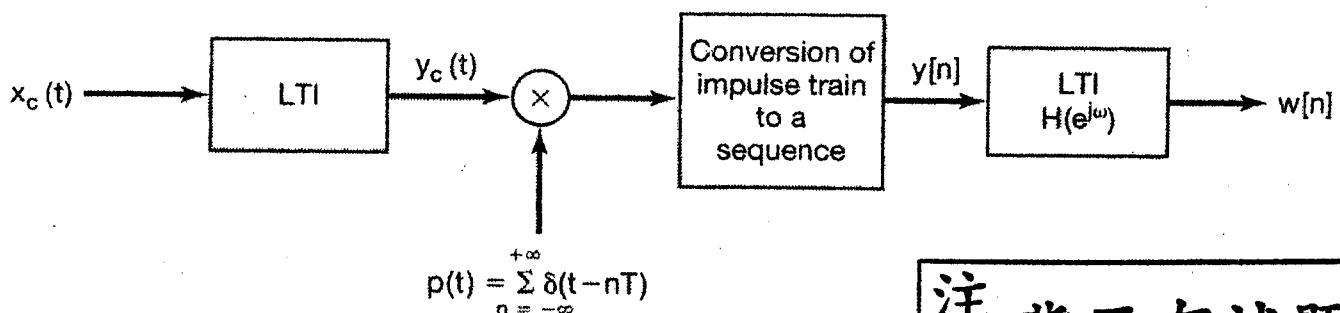You need to write down your answers only. No partial scores for your computation procedures.

四、(10%) The following figure shows a system consisting of a continuous-time LTI system followed by a sampler, conversion to a sequence, and an LTI discrete-time system. The continuous-time LTI system is causal and satisfies the linear, constant-coefficient differential equation

$$\frac{dy_c(t)}{dt} + y_c(t) = x_c(t).$$

The input $x_c(t)$ is a unit impulse $\delta(t)$.

(一) Determine $y_c(t)$. (5%)

(二) Determine the frequency response $H(e^{j\omega})$ and the impulse response $h[n]$ such that $w[n] = \delta(n)$. (5%)

注意：背面有試題

參考用

五、(5%) A signal x[n] has a Fourier transform $X(e^{j\omega})$ that is zero for $(\pi/4) \le |\omega| \le \pi$. Another signal

$$g[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n-1-4k]$$

is generated. Specify the frequency response $H(e^{j\omega})$ of a lowpass filter that produces x[n] as output when g[n] is the input.

六、(10%) Consider a continuous-time LTI system for which the input x(t) and output y(t) are related by the differential equation

$$\frac{d^2 y(t)}{dt^2} + \frac{dy(t)}{dt} - 2y(t) = x(t).$$

Suppose the system is stable. Determine y(t) as $x(t) = \sum_{n=1}^{\infty} u(t-n)$, where u(t) denotes the unit step function.

七、(10%) Consider a causal and stable LTI system with a real impulse response h(t) and system function H(s). Suppose that H(s) is rational, one of its poles is at -2+j, one of its zeros is at 5-2j, and it has exactly three zeros at infinity. For each of the following statements, determine whether it is true or false. Justify your answers.

(一) (2%) $h(t)e^{-t}$ is absolutely integrable.

(二) (2%) There exists a pole at 2 + j.

(三) (2%) The differential equation relating input x(t) and output y(t) for the system may be written in a form having only real coefficients.

(四) (2%) H(s) does not have fewer than five poles.

(五) (2%) If the input is $e^{5t}\sin(2t)$, the output is $e^{5t}\cos(2t)$.

八、(20%)

(一) (10%) Suppose we are given the following information about a continuous-time periodic signal x(t) with period 3 and Fourier coefficients a_k:

1. $a_k = a_{-k}$　　2. $a_k = a_{k+2}$　　3. $\int_{-1}^{1} x(t)dt = 2$　　4. $\int_{1}^{2} x(t)dt = 1$

Determine x(t) and the corresponding Fourier series representation.

(二) (10%) Consider the cascade interconnection of three LTI systems of the impulse responses $\sin(9\pi t)/\pi t$, $\sin(18\pi t)/\pi t$, and $\sin(27\pi t)/\pi t$, respectively. With x(t) obtained in (一) as the input, determine the Fourier series representation of the corresponding output.

九、(20%) Consider a discrete-time system with input x[n] and output y[n] for which

$$-\frac{1}{8}y[n-1] + \frac{1}{4}y[n] + y[n+1] = -2x[n-1] + x[n].$$

(一) (10%) Suppose all z with Re{z}>5 are in the region of convergence of the system function H(z). Determine H(z) and indicate the region of convergence. What is the impulse response?

(二) (10%) Draw three block diagrams for the system in the direct form, cascade form, and parallel form, respectively. Note that each block diagram should have the minimum number of delay elements.

注意：背面有試題