科目：訊號與系統(300B)　　校系所組：清華大學電機工程學系（乙組、丁組）

參考用

1. (10%) A discrete-time linear time-invariant (LTI) system has an impulse response $h[n]$ defined as follows:

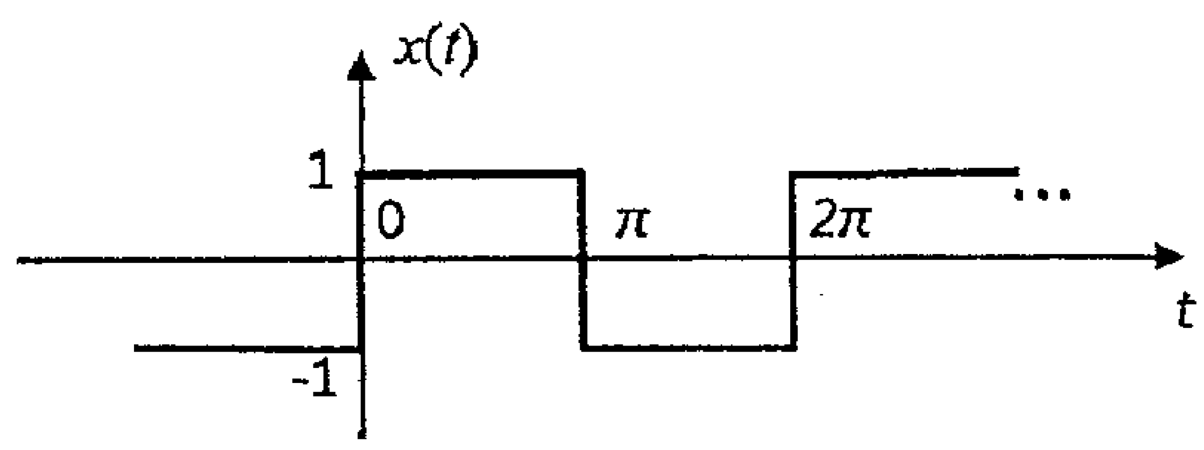
$$h(n) = \begin{cases} \cos^2\left(\frac{\pi}{2}\cdot\frac{n}{N}\right), & -N \le n \le N \\ 0 \quad , & \text{elsewhere.} \end{cases}$$

(a) (3%) Is the system causal?
(b) (3%) Is the system stable?
(c) (4%) Make a sketch of the unit step response of the system.

2. (10%) A square wave $x(t)$ has a period of 2π and its values are defined as follows,

$$x(t) = \begin{cases} 1, & 0 < t < \pi \\ 0, & t = \pi \\ -1, & \pi < t < 2\pi \end{cases}$$

and based on its period we assume that $x(t+2\pi) = x(t)$ for all t.



(a) (3%) Explain that $\int_{-\pi}^{\pi} x(t)\cos(kt)\,dt = 0$ for all integers k.

(b) (4%) Write the Fourier series of $x(t)$ as follows:

$$x(t) = \sum_{k=1}^{\infty} a_k \sin(kt), \text{ for } 0 < t < 2\pi$$

Find an expression for the coefficients a_k's. Note that you may need to consider odd and even k's separately.
(c) (3%) Use Parseval's theorem to calculate $\sum_{k=1}^{\infty} a_k^2$.

3. (10%) A second-order continuous-time linear dynamic system is characterized by the following equation:

$$\ddot{y} + 2\beta\dot{y} + \omega_0^2 y = \alpha x,$$

where $x(t)$ denotes the input, $y(t)$ denotes the output, and let us assume that $\beta > 0$ and $\beta^2 < \omega_0^2$.
(a) (3%) Make a sketch of the impulse response of the system.
(b) (3%) Calculate the transfer function $H(j\omega) \triangleq \frac{Y(j\omega)}{X(j\omega)}$.
(c) (4%) Make a sketch of the magnitude response of $H(j\omega)$. If $\beta^2 \ll \omega_0^2$, what is the approximate frequency at which the magnitude response reaches its maximum?

4. (10%) Determine all possible causal stable transfer functions $H(z)$ with a square-magnitude function given by

$$\left|H(e^{j\omega})\right|^2 = \frac{9(1.0625+0.5\cos\omega)(1.49-1.4\cos\omega)}{(1.36+1.2\cos\omega)(1.64+1.6\cos\omega)}$$

注意：背面有試題

5. (10%) A causal, nonideal lowpass filter is designed with frequency response $H(e^{jw})$. The difference equation relating the input $x[n]$ and output $y[n]$ for this filter is specified as

$$y[n]=\sum_{k=1}^{N} a_k y[n-k]+\sum_{k=0}^{M} b_k x[n-k]$$

The filter also satisfies the following specifications for the magnitude of its frequency response:

passband frequency = w_p

passband tolerance = δ_p

stopband frequency = w_s

stopband tolerance = δ_s

Now consider a causal LTI system whose input and output are related by the difference equation

$$y[n]=\sum_{k=1}^{N}(-1)^k a_k y[n-k]+\sum_{k=0}^{M}(-1)\ b_k x[n-k]$$

Show that this filter has a passband with a tolerance δ_p, and specify the corresponding location of the passband.

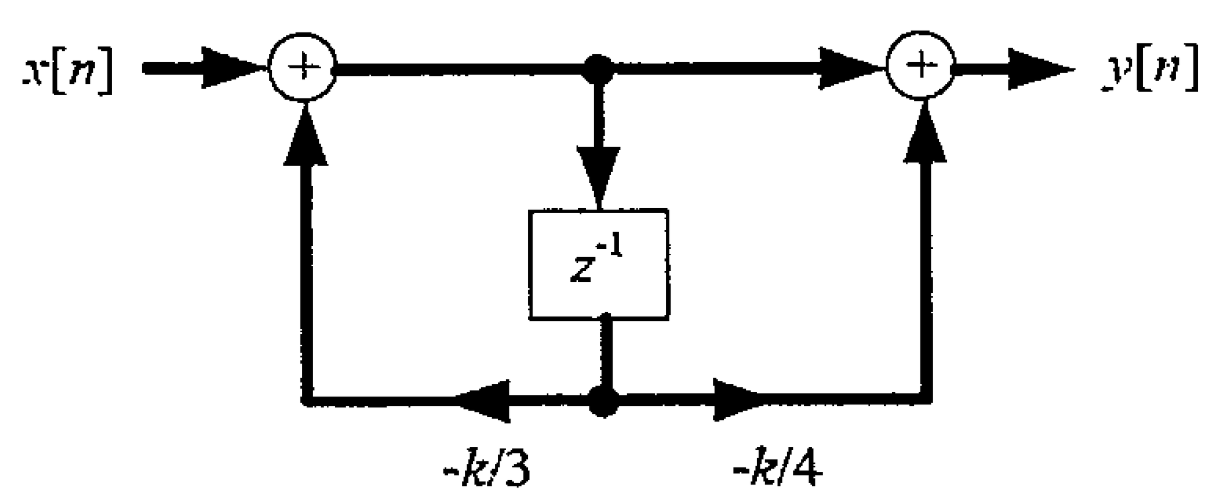
6. (10%) In many practical situations a signal $x(t)$ is recorded in the presence of an echo, which we would like to remove by appropriate processing as described below. Assume the echo is represented by an attenuated delay replication of signal $x(t)$. Thus the recorded signal is $s(t) = x(t) + \alpha x(t-T_0)$, where $|\alpha| < 1$. The recorded signal $s(t)$ is to be processed to recover $x(t)$ by first converting (ideally) to a sequence $s[n]$ (i.e., $s[n] = s(nT)$, T: sampling period), and then using an appropriate digital filter $h[n]$ to process $s[n]$ to get digital filter output $y[n]$. The output $y[n]$ is then converted to an impulse train $y_p(t)$ followed by an ideal lowpass filter with gain A and cutoff frequency π/T to get an analog output signal $y_c(t)$. Assume that $x(t)$ is band-limited (i.e., $X(jw) = 0$ for $|w| > w_M$) and that $|\alpha| < 1$. If $T_0 < \pi/w_M$, and the sampling period T is taken to be equal to T_0, determine the difference equation for the digital filter $h[n]$ and the gain A of the lowpass filter such that $y_c(t) = x(t)$.

7. (10%) Consider the causal digital filter structure shown below.

(a) (4%) Find $H(z)$ for this causal filter and its region of convergence.

(b) (2%) For what values of k is the system stable?

(c) (4%) Determine $y[n]$ if $k = 1$ and $x[n] = (2/3)^n$ for all n.



8. (10%) For a discrete-time LTI system, we know that if the input is

$$x[n]=\left(\frac{1}{3}\right)^n u[n]-\frac{1}{4}\left(\frac{1}{3}\right)^{n-1} u[n-1]$$

then the output is

$$y[n]=\frac{3}{2}\left(\frac{1}{2}\right)^n u[n]$$

(1) Find the corresponding impulse response $h[n]$ of the LTI system.

(2) Find a difference equation relating $x[n]$ and $y[n]$ that describes the LTI system.

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9. (10%) Consider a discrete-time system consisting of a parallel combination of N LTI filters with impulse response $h_k[n]$, $k=0,1,\dots,N-1$, that satisfy

$$h_k[n] = e^{j(2\pi nk/N)} h_0[n]$$

as shown in Figure A, and the frequency response of $h_0[n]$ is shown in Figure B, where $2\pi/\omega_c$ is an integer.

(1) Sketch the Fourier transforms of $h_1[n]$, $h_2[n]$, $h_{N-2}[n]$, $h_{N-1}[n]$.

(2) Find the minimum number of filters N required such that the overall system is an identity system.

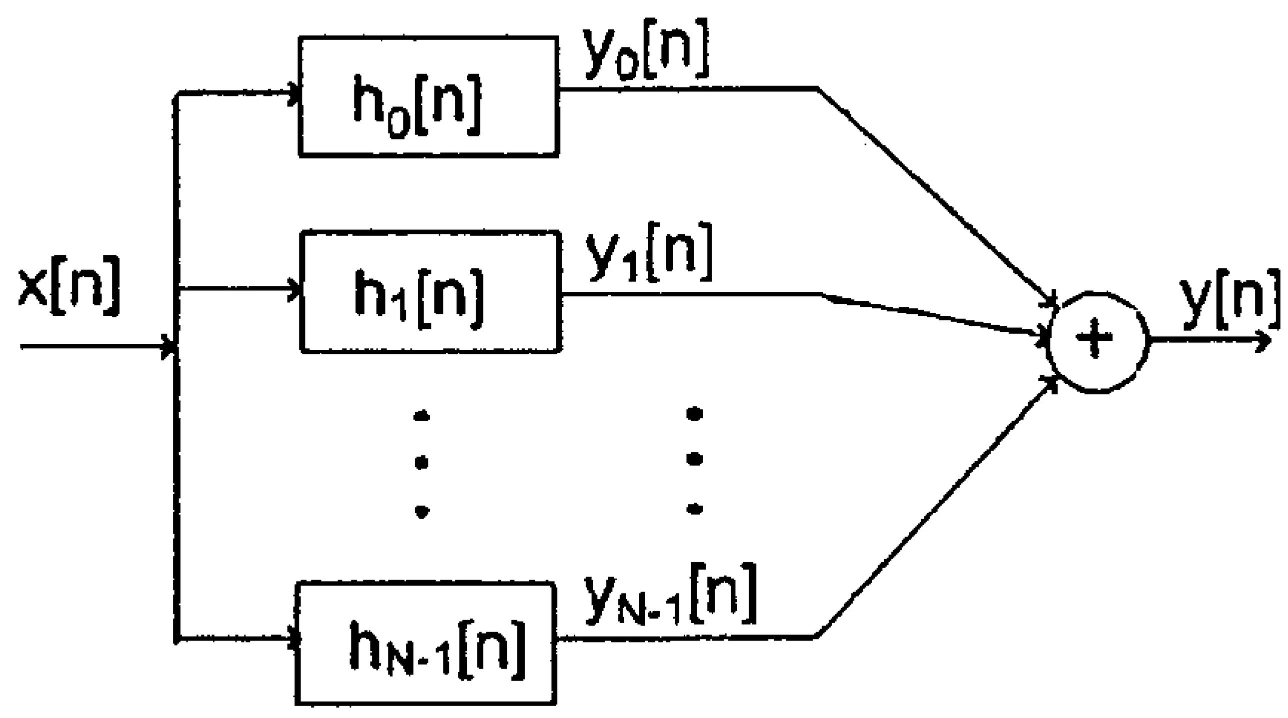


Figure A

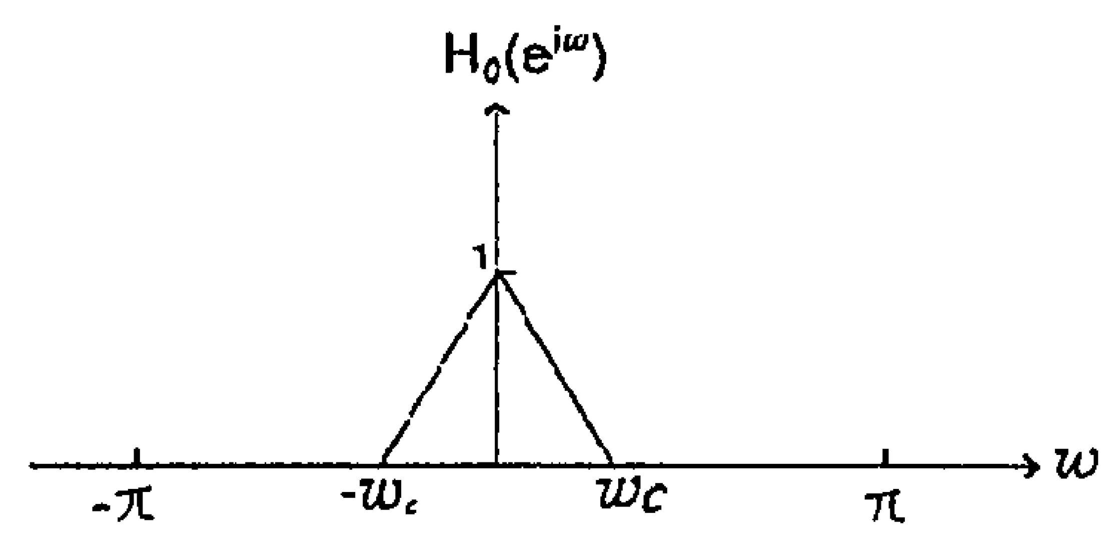
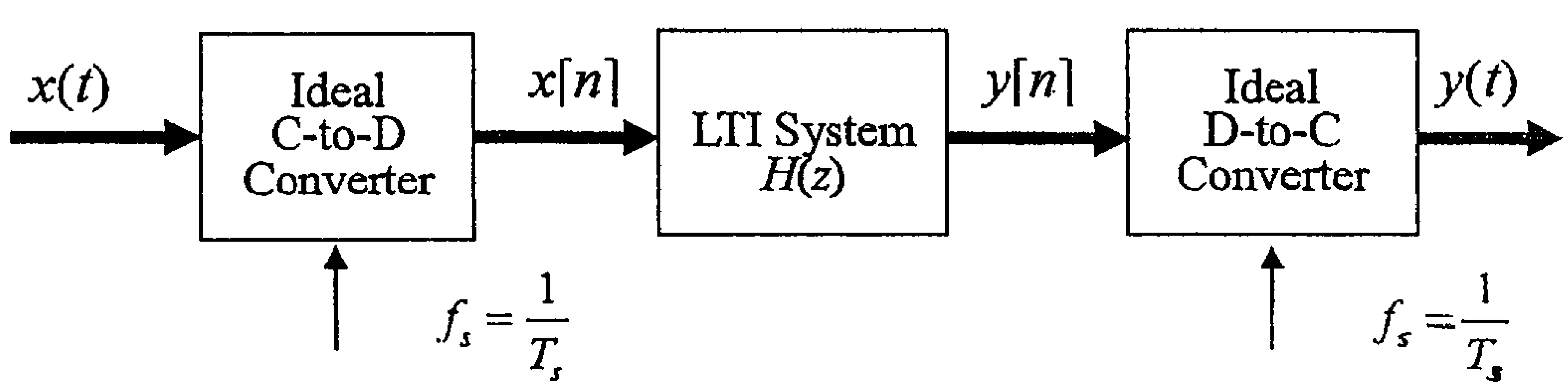


Figure B

10. (10%) The input to the C-to-D (Continuous-to-Discrete) converter in the following figure is

$$x(t) = 4 + \cos(500\pi t) - 3\cos[(2000\pi/3)t]$$



The system function for the LTI system is

$$H(z) = \frac{(1-z^{-1})(1-e^{j\pi/2}z^{-1})(1-e^{-j\pi/2}z^{-1})}{(1-0.9e^{j2\pi/3}z^{-1})(1-0.9e^{-j2\pi/3}z^{-1})}$$

If $f_s = 1000$ samples/sec, determine an expression for $y(t)$, the output of the D-to-C (Discrete-to-Continuous) converter.