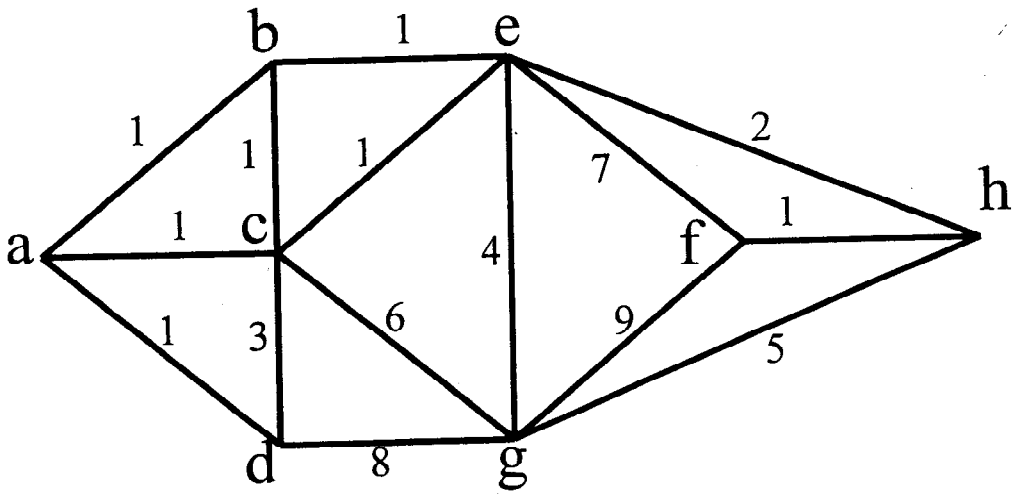
國 立 清 華 大 學 命 題 紙

99 學年度 資訊系統與應用研究所丙組(生物及醫學資訊組) 碩士班入學考試

科目 離散數學 科目代碼 2201 共 3 頁，第 1 頁 *請在【答案卷卡】作答

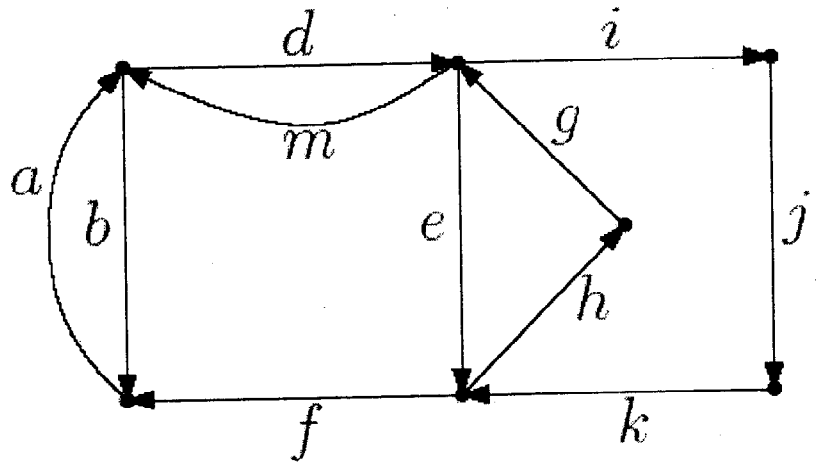
1. Answer the following short questions about trees.
   (a) (3%) Give the definition of a tree. (Please be as precise as you can.)
   (b) (3%) A binary tree has 300 leaves. How many internal nodes does this binary tree have?
   (c) (3%) Consider the problem of connecting 200 lamps to a single electric outlet by using extension cords each of which has 4 outlets. How many extension cords are needed?
   (d) (3%) A regular m-ary tree of height h has z leaves. What are the maximum and minimum values of z?
   (e) (3%) Draw the binary tree of which the preorder traversal is $a, b, d, g, e, h, i, c, f$, and the inorder traversal is $g, d, b, h, e, i, a, c, f$.
   (f) (3%) Suppose we are given the preorder traversal and the postorder traversal of a binary tree. Can we reconstruct the tree? If so, give an algorithm for doing so. If not, give a counterexample.
   (g) (3%) Use the procedure invented by Huffman to construct an optimal tree for a given set of weights: 1, 2, 3, 4, 5, 6, 7, 8.
2. (4%) Let I denote the sum of the path lengths of all the branch nodes and E denote the sum of the path lengths of the leaves in a rooted tree. Show that for a regular m-ary tree, $E = (m-1)I + mi$, where i is the number of branch nodes.
3. (4%) Determine a minimum spanning tree for the following graph.



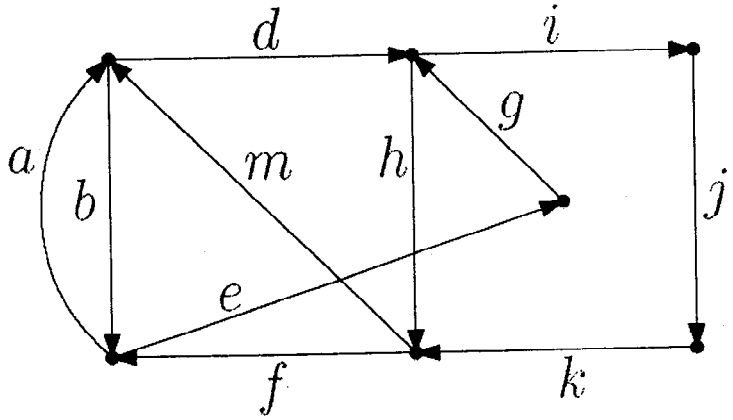
4. (4%) A three-state finite state machine has {0, 1} as its input and output alphabets. Given the following input sequence and its corresponding output sequence, determine the machine.

Input sequence	00010101
Output sequence	011001110

5. (9%) Let f an g be functions on {1, 2, 3, …}. Define $f(n)=O(g(n))$, $f(n)=\Omega(g(n))$, and $f(n)=\Theta(g(n))$, respectively.
6. (9%) Show that postage of 24 cents or more can be achieved by using only 5-cent and 7-cent stamps.
7. (8%) Show that for any set X, X is not equivalent to the power set of X.
8. (a) (4%) What is the order of $n!$ in the best big Oh notation as a function of n?
   (b) (4%) What is the order of lg (n!) in the best big Oh notation as a function of n?
9. (11%) Find the solution to $a_n = 2\,a_{n-1} + 5\,a_{n-2} - 6\,a_{n-3}$ with $a_0 = 7$, $a_1 = -4$, and $a_2 = 8$.
10. (5%) Consider the relation on the set of integers $R = \{(x, y) | x - y \text{ is an integer}\}$. Show that R is an equivalence relation.
11. (4%) Find the transitive closure of the relation {(1, 1), (1, 3), (2, 1), (2, 3), (2, 4), (3, 2), (3, 4), (4, 1)}.
12. Construct an Eulerian tour or open Eulerian trail for the following graphs, respectively.
    (a) (4%)



(b) (4%)



13. (5%) The coloring number of an acquaintance network tells the minimum number of groups into which the persons in that network must be partitioned so that no two persons in a group have prior acquaintance. Calculate the coloring number of the following acquaintance network.