

97 學年度 資訊系統與應用研究所 系(所) 丁 組碩士班入學考試

科目 機率論 科目代碼 2402 共 4 頁第 1 頁 *請在試卷【答案卷】內作答

I. (25%) Please answer the following questions.

1. (15%) A pair of dice is rolled n times.
 - (a) (5%) Find the probability that “seven” will show at least once.
 - (b) (5%) Find the probability that double six will not show at all.
 - (c) (5%) Find the probability of obtaining double six at least once.

2. (10%) Consider the coin experiment where the probability of “head” equals p and the probability of “tail” equals $1 - p$. If we toss the coin till a head appears for the first time, what is the probability that the number of required tosses is odd?

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II. (25%) Answer the following questions.

1. (5%) Let Y have a uniform distribution $U(0, 1)$, and let

$$X = a + (b - a)Y, \quad a < b.$$

Find the distribution function $F(x) = P(X \leq x)$.

2. Customers arrive randomly at a bank teller's window. Given that one customer arrived during a particular 10-minute period, let X equal the time within the 10 minutes that the customer arrived. Suppose X has the uniform distribution $U(0, 10)$.

- (a) (5%) Find the mean of X .
 (b) (5%) Find the variance of X .

3. Let X equal the weight (in grams) of eggs. Assuming that the distribution of X is the normal distribution given by $N(55, 100)$, find

- (a) (5%) $P(57.5 \leq X \leq 67.5)$.
 (b) (5%) $P(47.5 \leq X \leq 60)$.

[Hint:] The following function values may be useful for your computation.

$$P(Z \leq z) = \Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-w^2/2} dw$$

$$\Phi(0.25) = 0.5987, \quad \Phi(0.5) = 0.6915, \quad \Phi(0.75) = 0.7734, \quad \Phi(1) = 0.8413, \quad \Phi(1.25) = 0.8944, \\ \Phi(1.5) = 0.9332$$

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III. (25%) Answer the following questions.

1. (10%) A particular disease is known to be found in men over 65 with probability 20%. A blood test has been used to detect this disease with a 6% false negative (i.e., the test incorrectly gives a negative result) rate and a 3% false positive (i.e. the test incorrectly gives a positive result) rate. Note that the positive result means the disease is found in the test, while the negative result means the disease is not found in the test.
 - (a) What is the probability that a man over 65 receives a positive test result?
 - (b) If a 70-year-old man took the test and received a positive result, what is the probability that he really has this disease?

2. (15%) The probability that a certain kind of electronic device is defective is 0.1. An inspector randomly picks 20 items from a shipment of this type of electronic device. Assume each test of a randomly selected item is a Bernoulli trial. Let a random variable X denote the total number of defective items in these 20 items.
 - (a) Write down the probability distribution of the random variable X .
 - (b) Compute the probability $P(X < 2)$.
 - (c) Derive the moment generating function for the random variable X . Show your derivation.

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IV. (25%) Please answer the following questions.

1. (13%) Let the random variable, X , have the moment generating function $M(t) = e^{3t+2t^2}$.

(a) Find the mean and variance of X , respectively.

(b) Give the probability density function of X .

(c) Let $Y=(X-3)/2$, how is Y distributed?

(d) Let $Z=Y^2$, how is Z distributed?

2. (12%) Let X equal the number of bad records in each 100 feet of a used computer tape. Assume that X has a Poisson distribution with mean 2.5. Let W equal the number of feet before the first record is found.

(a) Give the mean number of flaws per foot.

(b) How is W distributed?

(c) Give the mean and variance of W .

(d) Find $P(W \leq 20)$ and $P(W > 40)$.