I. (25%)　Answer the following questions.

1.　(15%) A pair of dice is rolled n times.
   (a) (5%) Find the probability that "seven" will not show at all.
   (b) (5%) Find the probability that "seven" will show at least once.
   (c) (5%) Find the probability of obtaining double six at least once.

2.　(10%) Consider the coin experiment where the probability of "head" equals p and the probability of "tail" equals $1-p$. If we toss the coin till a head appears for the first time, what is the probability that the number of required tosses is odd?

II. (25%) Answer the following questions.

1. The random variable x has the probability distribution

$$f(x) = e^{-x} \qquad (0 \le x < \infty)$$

   (a) (5%) Determine the probability that a measurement of x will be greater than the mean of x.

   (b) (5%) Find the variance of x.

   (c) (5%) Two values x_1 and x_2 are chosen independently. Find the expected value of $x_1 + x_2$

2. Suppose that n measurements of the random variable x yield the results $x_1, x_2, \ldots, x_n$, and that x is assumed to have a normal distribution

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

   with the two unknown parameters μ and σ^2.

   (a) (5%) Give a maximum likelihood estimate for σ^2 in terms of the data $x_1, \ldots, x_n$.

   (b) (5%) An estimate of a parameter is said to be unbiased if the expected value of the estimate equals the parameter. Show that the maximum likelihood estimate of μ is unbiased.

III. (25%)　Answer the following questions.

1. (10%) A particular disease is known to be found in men over 65 with probability 20%. A blood test has been used to detect this disease with a 6% false negative (i.e., the test incorrectly gives a negative result) rate and a 3% false positive (i.e. the test incorrectly gives a positive result) rate. Note that the positive result means the disease is found in the test, while the negative result means the disease is not found in the test.

   (a) What is the probability that a man over 65 receives a positive test result?

   (b) If a 70-year-old man took the test and received a positive result, what is the probability that he really has this disease?

2. (15%) The probability that a certain kind of electronic device is defective is 0.1. An inspector randomly picks 20 items from a shipment of this type of electronic device. Assume each test of a randomly selected item is a Bernoulli trial. Let a random variable X denote the total number of defective items in these 20 items.

   (a) Write down the probability distribution of the random variable X.

   (b) Compute the probability $P(X < 2)$.

   (c) Derive the moment generating function for the random variable X. Show your derivation.

IV. (25%)　Please answer the following questions.

1. (7%) A random sample of size 16 from the normal distribution with the mean μ and variance 25 yielded the estimated mean $\bar{\mu}$ =73.8. Find a 95% confidence interval for μ.

2. (8%) The length in centimeters of n=29 fish yielded an average length of $\bar{x}$ =16.82 and s^2 =34.9. Determine the size of a new sample so that [$\bar{x}$ -0.5, $\bar{x}$ +0.5] is an approximate 95% confidence interval for the mean.

3. (10%) Let $W_1 < W_2 < \cdots < W_n$ be the order statistics of a random sample of size n from the uniform distribution U(0,1).

   (a) Find the probability density function of W_1.

   (b) Find the probability density function of W_n.

   (c) Use the results of (a) to find $E(W_1)$.

   (d) Use the results of (b) to find $E(W_n)$.