# 國 立 清 華 大 學 命 題 紙

97 學年度 資訊系統與應用研究所 （所） 丙 組碩士班入學考試

科目 離散數學 科目代碼 2301 共 2 頁第 1 頁 *請在【答案卷卡】內作答

1. (10%) If $f(n) = 2f(4n^{1/2}) + \log_2 n$, then, what is $f(n)$ ( as the big $O$ function of $n$ )?

2. (10%) A relation R that is reflexive, symmetric, and transitive is said to be an equivalence relation. Show that the following relation is an equivalence relation.

The relation R on integer defined by $i$R$j$ if and only if $i = j$.

3. (10%) What is wrong with the following induction "proof" that all elements in any set are identical. If the set only has one element, then it is true. Assume that it is true for the set with $k$-1 elements. Now, for a set $S$ with $n$ elements, let $S = A \cup B$, here $A$ and $B$ each have $k$-1 elements and element $a$ is in $A$ and in $B$. By the induction hypothesis, all element in $A$ are identical to $a$, and all element in $B$ are identical to $a$. Therefore, all elements in $S$ are identical to $a$.

4. (14%) Let $p$ and $q$ be the propositions in the following.

$p$: You drive over 65 miles per hour.

$q$ : You get a speeding ticket.

Write the following propositions using $p$ and $q$ and logical connectives.

(a) You do not drive over 65 miles per hour.

(b) You drive over 65 miles per hour, but you do not get a speeding ticket.

(c) You will get a speeding ticket if you drive over 65 miles per hour.

(d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

(e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

(f) You get a speeding ticket, but you do not drive over 65 miles per hour.

(g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

5. (10%) There are 10 copies of one book and one copy each of 10 other book. In how many ways can we select 10 books?

6. (10%) How many subsets of a (2$n$+1)-element set have $n$ elements or less?

7. (10%) How many integers between 1 and 1,000,000 have the sum of the digits equal to 20?

8. (10%) Solve the recurrence relation $\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}}$ with initial conditions $a_0 = a_1 = 1$.

9. (16%) Determine whether each of the following implications is true or false.

(a) If 1+1=2, then 2+2=5.

(b) If 1+1=3, then 2+2=4.

(c) If 1+1=3, then 2+2=5.

(d) If pigs can fly, then 1+1=3.

(e) If 1+1=3, then God exists.

(f) If 1+1=3, then pigs can fly.

(g) If 1+1=2, then pigs can fly.

(h) If 2+2=4, then 1+2=3.