

國 立 清 華 大 學 命 題 紙

97 學年度 _____ 資訊系統與應用研究所 _____ 系 (所) _____ 甲 _____ 組碩士班入學考試

科目 _____ 工程數學 _____ 科目代碼 _____ 2102 _____ 共 4 頁第 1 頁 *請在試卷【答案卷】內作答

I. (25%) Answer the following questions.

1. (5%) Let

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \text{ and } B = \begin{bmatrix} 4a + 5d + 6g & 4b + 5e + 6h & 4c + 5f + 6i \\ 2a + 3d & 2b + 3e & 2c + 3f \\ a & b & c \end{bmatrix}.$$

If the determinant of A is 5, what is the determinant of B ?

2. (10%) Consider the linear system $Ax = b$ given by

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 5 \\ 4 & 1 & s \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ t \end{bmatrix}$$

If the matrix A is **not** invertible, what is the value of s ? Apply this value of s and find the value of t that makes the linear system $Ax = b$ have a solution.

3. (10%) For the linear transformation $T : R^3 \rightarrow R^2$ with $T(a_1, a_2, a_3) = (a_1 - 2a_2, 3a_3)$, find a basis for the null space of T , and compute the nullity and rank of T .

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II. (25%) Answer the following questions.

1. (15%) Given a matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}.$$

- (a) (10%) Use the Gram-Schmidt process to factor A into a product QR , where $Q = (\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$ is a matrix with orthonormal column vectors $\{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3\}$ and R is an upper triangular matrix with positive diagonal entries.
- (b) (5%) If $\mathbf{x} = 2\mathbf{q}_1 + 2\mathbf{q}_2 + \mathbf{q}_3$ and $\mathbf{y} = 5\mathbf{q}_1 + \mathbf{q}_3$, determine the vector norm $\|\mathbf{x}\|_2$ and the inner product $\langle \mathbf{x}, \mathbf{y} \rangle$.

2. (10%) Suppose that a 3×3 matrix A has eigenvalues $\lambda = 0, -1, \text{ and } 1$ with the corresponding eigenvectors $(0, 1, -1)^T, (0, 1, 1)^T, (1, -1, 1)^T$, respectively.

- (a) (5%) Find the matrix A .
- (b) (5%) Compute A^k for an arbitrary positive integer k .

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科目 工程數學 科目代碼 2102 共 4 頁第 3 頁 *請在試卷【答案卷】內作答

III. (25%) Answer the following questions.

1. (10%) Consider a continuous random variable X with a normal (Gaussian) distribution with mean 10 and variance 25.
 - (a) Determine the probability $P(X > 20)$. Write your answer in terms of the cumulative distribution function Φ of a standard normal random variable Z , i.e. $\Phi(z) = P(Z \leq z)$.
 - (b) For a random variable Y defined by $Y = 2X + 5$, what is the probability density function for the random variable Y ? What are the mean and variance for Y ?

2. (15%) The joint probability density function for two continuous random variables X and Y is given as follows:

$$f(x, y) = \begin{cases} ce^{-(2x+y)} & 0 < x < 1 \text{ and } 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

where c is a constant.

- (a) Determine the constant c so that it satisfies the property of a joint probability density function.
- (b) Compute the expectation $E(X)$. Show the detailed steps of your derivation.
- (c) Compute the conditional probability $P(X > 0.5 \mid Y < 1)$. Show the detailed steps of your derivation.

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IV. (25%) Answer the following questions.

1. (6%) Let X have a geometric distribution.

(a) Give the probability density function $P(X=x)$ of X .

(b) Show that $P(X>k+j | X>k) = P(X>j)$, where k and j are any nonnegative integers.

2. (6%) Let Y have a binomial distribution with mean 6 and variance 3.

(a) Give the probability density function $P(Y=y)$ of Y .

(b) Find $P(Y=2)$.

3. (13%) Let W have a Poisson distribution with the variance 3.

(a) Give the probability density function $P(W=w)$ of W .

(b) Find the moment-generating function $M(t)$ of W .

(c) Find $P(Y \geq 2)$.