95 學年度 資訊系統與應用研究所 系（所） 丙 組碩士班入學考試

科目 機率與統計 科目代碼 2402 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

I. (15%) Suppose box I contains 2 white balls and 1 black ball, box II contains 1 white ball and 3 black balls, and box III contains 2 white balls and 2 black balls.

(a) (5%) One ball is selected from each box. Calculate the probability of getting all white balls.

(b) (5%) One box is selected at random and one ball is drawn from it. Calculate the probability that it will be white.

(c) (5%) In (b), calculate the probability that the first box was selected given that a white ball is drawn.

II. (15%) Let X and Y be independent random variables each having the uniform density on $\{0,1,\cdots,N\}$.

(a) (5%) Find $P(X \geq Y)$.

(b) (5%) Find $P(X = Y)$.

(c) (5%) Find the density of $\min(X,Y)$.

III. (10%) The joint probability density function for random variables X and Y is given by

$$f(x,y)=\begin{cases} cxy & 0\le x\le 3, 0\le y\le 2 \\ 0 & elsewhere \end{cases}$$

where c is a constant.

(a) (5%) Determine the constant c so that it satisfies the property of joint probability density function.

(b) (5%).Are the random variables X and Y dependent? Give the reason for your answer.

IV. (10%) Consider a continuous random variable X with a normal distribution given as follows:

$$f(x)=\frac{1}{2\sqrt{2\pi}}e^{-\frac{(x-1)^2}{8}}$$

(a) (5%) Find a linear transformation on the random variable X to make the transformed random variable to have the standard normal distribution.

(b) (5%) Compute the probability P(0 < X < 3). Write your answer in terms of the cumulative distribution function Φ of a standard normal random variable Z, i.e. $\Phi(z)=P(Z\le z)$.

V. (15%) Assume each observation X_i, i=1, …, 100, is randomly drawn from a population with a continuous uniform distribution in [0, 2].

(a) (5%) Compute the mean and variance for the sample mean $\bar{X}$.

(b) (5%) Give an approximate distribution for $\bar{X}$ as best as you can. Explain your reason.

(c) (5%) Derive the probability that the sample mean value is larger than 1.2. You can write your answer in terms of some cumulative distribution function.

VI.(35%) Answer the following questions.

1.(20%) Let $X_1, X_2, \ldots, X_n$ be a random sample from $b(1,p)$, thus $Y = \sum_{i=1}^{n} X_i \sim b(n,p)$.

(a) (5%) Show that $\overline{X} = Y/n$ is an unbiased estimator of p.

(b) (5%) Show that $Var(\overline{X}) = p(1-p)/n$.

(c) (5%) Show that $E[\overline{X}(1-\overline{X})] = \frac{n-1}{n}[p(1-p)]$.

(d) (5%) Find the value c so that $c\overline{X}(1-\overline{X})$ is an unbiased estimator of $Var(\overline{X})$.

2.(8%) To test the equal likelihood of the six faces of a die, the die was cast 120 times with the results listed in the following table.

Face Value	1	2	3	4	5	6
Frequency	18	23	16	21	18	24

Let X_k have χ^2 distribution with the degree of freedom k, $k \geq 1$ and assume that $P(X_5 > 11.07) = 0.05$ and $P(X_6 > 12.59) = 0.05$. By applying a Pearson's χ^2 goodness-of-fit test, to see if this die is a fair die, what is the χ^2 statistic? how do you draw your conclusion?

3.(7%) Observe a set of n points $S_n = \{(x_i, y_i) | \ 1 \leq i \leq n\}$. A best line fit problem is to find α, β to minimize

$$f(\alpha, \beta) = \sum_{i=1}^{n} (y_i - \alpha - \beta x_i)^2$$

Given $S_3 = \{(0,1), (3,4), (5,6)\}$, find a best fitting line for S_3 and the corresponding R^2 statistic.