國　立　清　華　大　學　命　題　紙

96 學年度 資訊系統與應用 系（所） 丙 組碩士班入學考試

科目 離散數學 科目代碼 2401 共 3 頁第 1 頁 *請在【答案卷卡】內作答

1. (15%) The *distance* between two vertices in an undirected graph is defined to be the minimum number of edges in a path between them. The *diameter* of an undirected graph is defined to be the maximum of the distances between all pairs of vertices.

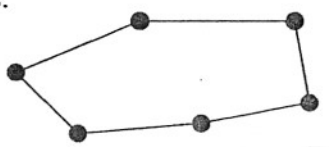


Figure 1.

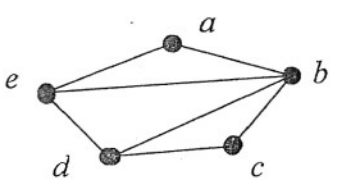
(a) (3%) What is the diameter of the graph in Fig. 1?

(b) (6%) Let d denote the diameter of a graph with n vertices. Let k denote the maximum of the degrees of the vertices. Show that

$$1 + k + k(k-1) + k(k-1)^2 + \cdots + k(k-1)^{d-1} \geqq n .$$

(c) (3% + 3%) What is the minimum diameter of a graph with **ten** vertices that are all of degree **three**? Draw an example of such a graph.

2. (6%) A set of vertices in an undirected graph is said to be an *independent set* if no two vertices in it are adjacent. A *maximal independent set* is an independent set which will no longer be one when any vertex is added to the set. Find all maximal independent sets of the graph in Fig.2.



3. (6%) Show that among $n + 1$ positive integers less than or equal to $2n$ there are two of them that are relatively prime.

4. (10%) The *undirected complete graph* of n vertices, denote K_n, is a graph with n vertices in which there is an edge between each pair of distinct vertices.

   (a) (2%) How many edges does K_6 have?

   (b) (8%) If the edges of K_6 are painted either red or blue, prove that there is a red triangle or a blue triangle that is a subgraph of K_6.

5. (13%) By traversing a tree, we mean to visit each of the vertices of the tree exactly once in some sequential order. We describe here three principal ways that may be used to traverse a *binary tree*: i) *preorder traversal*: visit the root, traverse the left subtree, and then traverse the right subtree; ii) *inorder traversal*: traverse the left subtree, visit the root, and then traverse the right subtree; iii) *postorder traversal*: traverse the left subtree, traverse the right subtree, and then visit the root.

   (a) (5%) Suppose we are given the preorder traversal and the postorder traversal of a binary tree. Can we reconstruct the tree? If so, give an algorithm for doing so. If not, give a counterexample.

   (b) (8%) Draw the binary tree of which the preorder traversal is $a, b, d, g, e, h, i, c, f$, and the inorder traversal is $g, d, b, h, e, i, a, c, f$.

6. (50%) Answer the following short questions. (You don't need to explain how you derive the answers.)

(a) (4%) What is the number of three-digit decimal numbers that contain no repeated digits?

(b) (4%) What is the number of different outcomes when three dice are rolled?

(c) (4%) How many set of solutions are there to the inequality $x_1+x_2+x_3 <11$, where all the unknowns are integers satisfying $x_1 > 0$, $x_2>1$, and $x_3>2$?

(d) (4%) Give a recurrence relation for the number of bit strings of length that do not have two consecutive 0s. (A bit string contains elements of 0s and 1s only.)

(e) (4%) Among the integers 1-100, how many of them are divisible by 3 or 5 or 7?

(f) (4%) Give two different but equivalent definitions of a tree.

(g) (4%) What is the sufficient and necessary condition for an directed graph to possess an Eulerian path?

(h) (4%) What is the relationship between i, the number of branch nodes, and t, the number of leaves, of a regular m-ary tree?

(i) (4%) A regular m-ary tree of height h has at least x leaves and at most y leaves. What are x and y?

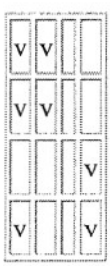
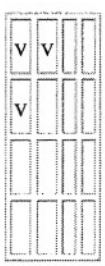
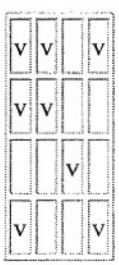
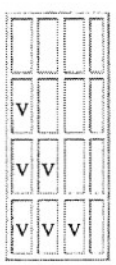
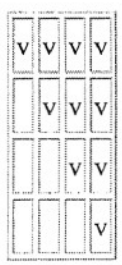
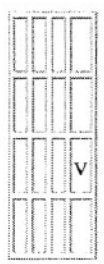
(j) (4%) Construct an optimal binary prefix code for a set of weights {8, 9, 12, 14, 16, 19}. (Please draw the corresponding binary tree explicitly.)

(k) (10%) Consider the following 7 binary relations on a set of size 4.

甲、 (2%) Which relations are symmetric?

乙、 (4%) Which relations are antisymmetric?

丙、 (4%) Which relations are transitive?

$R_1=$  , $R_2=$  , $R_3=$  , $R_4=$  , $R_5=$  , $R_6=$  , $R_7=$