95 學年度 資訊系統與應用研究所 系（所） 乙 組碩士班入學考試

科目 機率論 科目代碼 2302 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

I. (20%) A box contains 3 white balls and 2 black balls. Two balls are drawn from it without replacement.

(a) (5%) Calculate the probability that both balls drawn will be the same color.

(b) (5%) Calculate the probability that at least one of the balls drawn will be white.

(c) (5%) Calculate the probability that the second ball is black given that the first ball is black.

(d) (5%) Calculate the probability that the first ball is white given that the second ball is white.

II. (15%) Let X and Y be independent random variables each having the uniform density on $\{0,1,\cdots,N\}$.

(e) (5%) Find $P(X \geq Y)$.

(f) (5%) Find $P(X = Y)$.

(g) (5%) Find the density of $\min(X,Y)$.

III.(30%)

1. (10%) Denote $C_k^n = \frac{n!}{(n-k)!k!}$, $1 \le k \le n$ and define $0! = 1$.

(a) **(6%)** Show that $C_k^n = C_k^{n-1} + C_{k-1}^{n-1}$, $\forall\, 1 \le k \le n$.

(b) **(2%)** Find $\sum_{k=0}^{n} C_k^n$.

(c) **(2%)** Find $\sum_{k=0}^{n} (-1)^k C_k^n$.

2.(10%) Let X be the number of accidents in a factory per week having the probability mass function

$$f(x) = \frac{1}{(x+1)(x+2)}, \quad x = 0, 1, 2, \cdots,$$

Find the conditional probability $P(X \ge 4 | X \ge 1)$.

3.(10%) The p.d.f. of X is $f(x) = \theta x^{\theta-1}$, $0 < x < 1$, $0 < \theta < \infty$ and let $Y = -2\theta ln(X)$. Name the distribution of Y and find the expectation $E(Y)$ and variance $Var(Y)$.

IV. (10%) Consider two independent random variables X and Y satisfying the following equations:

$$E[(X+1)^2] = 9, \quad E[(X-3)^2] = 1$$
$$E[(Y-3)^2] = 24, \quad E[(Y+2)^2] = 9$$

(a) (5%) Compute the means and variances of X and Y.

(b) (5%) Compute the covariance between the random variables X+2Y and X-3Y.

V. (15%) The joint probability density function for random variables X and Y is given by

$$f(x,y) = \begin{cases} cxy & 0 \le x \le 3, 0 \le y \le 2 \\ 0 & elsewhere \end{cases}$$

where c is a constant.

(a) (5%) Determine the constant c so that it satisfies the property of joint probability density function.

(b) (5%) Compute the expectation E(X).

(c) (5%) Are the random variables X and Y dependent? Give the reason for your answer.

VI. (10%) Consider a continuous random variable X with a uniform distribution between 0 and 4.

(a) (5%) Compute the mean and variance of random variable X from their definitions.
Show the step-by-step details of your computation.

(b) (5%) Compute the moment generating function for the random variable X.