95 學年度 資訊系統與應用研究所 系（所） 甲 組碩士班入學考試

科目 工程數學 科目代碼 2202 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

I. (20%) Let $T: R^3 \to R^3$ be the linear transform defined by

$$T(\mathbf{x}) = (x_1 - x_3, -2x_1 + 3x_2 - x_3, 3x_1 - 3x_2)^T.$$

(a) (5%) Find the standard matrix representation A for the linear operator T.

(b) (5%) Find the LU decomposition of A, where L is a unit lower triangular matrix and U is an upper triangular matrix.

(c) (5%) Find a basis for the column space of A.

(d) (5%) Find a basis for the nullspace of A.

II. (15%) Let $V = span\{(2,0,-1,1)^T, (1,1,0,1)^T\}$.

(a) (5%) Find an orthonormal basis for V.

(b) (5%) Let $\mathbf{b} = (1,1,-3,1)^T$. Use your answer to (a) to find the projection $\mathbf{p}$ of $\mathbf{b}$ onto V.

(c) (5%) Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}$. Find the least squares solution of $A\mathbf{x} = \mathbf{b}$.

III.(15%) Let $A = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

1.(3%) Find the characteristic polynomial of matrix A.

2.(3%) Find the eigenvalues and corresponding orthonormal eigenvectors of A.

3.(3%) Give the spectrum decomposition for A.

4.(3%) Find the singular values of A.

5.(3%) Find $A^3 + 3A^2 - 13A - 17I$.

IV.(15%)

1.(8%) Let $f(x) = \frac{1}{2}x^2e^{-x},\ 0 < x < \infty$ be the p.d.f. of X.

(a) Find the moment-generating function $M(t)$ of X.

(b) Find the mean $E(X)$ and variance $Var(X)$.

2.(7%) The p.d.f. of X is $f(x) = \theta x^{\theta-1},\ 0 < x < 1,\ 0 < \theta < \infty$ and let $Y = -2\theta ln(X)$. Name the distribution of Y and find the expectation $E(Y)$ and variance $Var(Y)$.

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V. (10%) An industrial firm uses three different warehouses (A, B, and C) to store its manufactured product. From past records, it is known that 20% of the manufactured product are assigned to warehouse A, 50% are assigned to B, and 30% to warehouse C. It is known that 4% of the product in warehouse A are defective, 2% in warehouse B are defective, and 3% in warehouse C are defective.

(a) (5%) Assume a product is selected at random from one of these warehouses. What is the probability that it is defective?

(b) (5%) For a randomly selected defective product made by this firm, which warehouse is the most probable warehouse that stored it? Why?

VI. (10%) In the transmission of digital data over a wireless network, the probability that a bit is transmitted and received with error is 0.00001. Assume the probability of error occurring at one bit is independent of the probability of error occurring at another bit. Consider transmitting a file of size 200K bits. A random variable X denotes the total number of error bits in the received file.

(a) (5%) What probability distribution function is best suited for the random variable X? Write down the probability distribution function for X. State your reason.

(b) (5%) Give the probability when there are less than 2 errors in the received file. Try to simplify your answer as best as you can.

VII. (15%) The joint probability density function for random variables X and Y is given by

$$f(x,y)=\begin{cases} cxy & 0\le x\le 3, 0\le y\le 2 \\ 0 & elsewhere \end{cases}$$

where c is a constant.

(a) (5%) Determine the constant c so that it satisfies the property of joint probability density function.

(b) (5%) Compute the expectation E(X).

(c) (5%) Are the random variables X and Y dependent? Give the reason for your answer.