國　立　清　華　大　學　命　題　紙

96 學年度 資訊系統與應用 系（所） 甲 組碩士班入學考試

科目 基礎計算機科學 科目代碼 2201 共 3 頁第 1 頁 *請在【答案卷卡】內作答

1 (10%) Given a tree with *n* vertices,

(a) (3%) What is the maximum number of edges in this tree?

(b) (3%) What is the minimum number of edges in this tree?

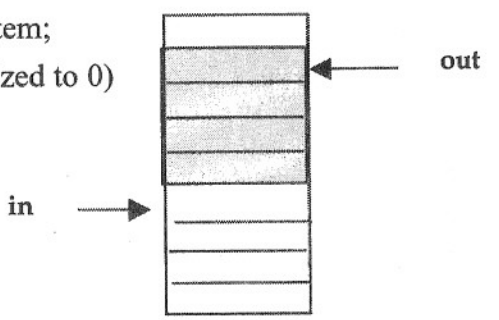
(c) (4%) Please prove your answer.

2. (a) (5%) Explain why it can only use BUFFER_SIZE-1 elements in the following program.

Implement the buffer as a circular array:

var *buffer*: array[1..*n*] of item;
in, *out*: 0..*n*-1; (initialized to 0)

next free: *in*
first available: *out*
empty: *in*=*out*



(b) (5%) If you can add one more variable to your program, what variable should you add in order to use all buffer elements. Please explain your answer.

3. (5%) Draw the diagram of hardware address protection, including the base and limit registers.

4. (10%) What is the difference between "Software Engineering" and "Capability Maturity Model for Software (CMM)"? Please give specific examples to illustrate your answer.

5. (15%) An assertion is a logical proposition that can be true or false. We can make assertions about the state of the developed program. The general concept behind formal program verification is that we can make assertions about what the program is intended to do, based on its specifications, and then prove through a logical argument that a design or implementation satisfies the assertions.

(a) (8%) Please explain the meaning of following program.

```
#include <assert.h>
cin >> newValue;
assert(cin);
inFile.open("data.in");
assert(inFile);
```

(b) (7%) Although executable assertions can be valuable tools, there is one serious drawback to their use. Please explain it.

6. (25%) Answer the following short questions. (You don't need to explain how you derive the answers.)

(a) (4%) How many set of solutions are there to the inequality $x_1+x_2+x_3 <11$, where all the unknowns are integers satisfying $x_1 > 0$, $x_2>1$, and $x_3>2$?

(b) (4%) Give a recurrence relation for the number of bit strings of length that do not have two consecutive 0s. (A bit string contains elements of 0s and 1s only.)

(c) (4%) What is the relationship between i, the number of branch nodes, and t, the number of leaves, of a regular m-ary tree?

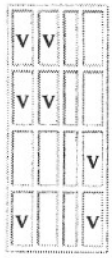
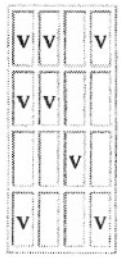
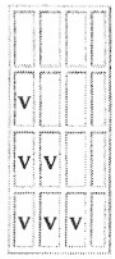
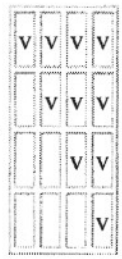
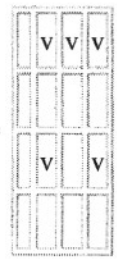
(d) (4%) A regular m-ary tree of height h has at least x leaves and at most y leaves. What are x and y?

(e) Consider the following 7 binary relations on a set of size 4.

(i) (1%) Which relations are symmetric?

(ii) (4%) Which relations are antisymmetric?

(iii) (4%) Which relations are transitive?

R_1=  , R_2=  , R_3=  , R_4=  , R_5=  , R_6=  , R_7= 

7. (10%) Suppose that we are given a set $\{a_1, a_2, \ldots, a_n\}$ of n positive integers, such that $a_1 + a_2 + \cdots + a_n < 2^n - 1$.
   (a) (2%) What is the number of subsets of $\{a_1, a_2, \ldots, a_n\}$?
   (b) (6%) Show that there must be two different subsets which have the same sum.
   (c) (2%) For the set {47, 59, 91, 100, 88, 111, 23, 133, 157, 205} (notice that their sum, 1014, is less than $2^{10} - 1 = 1023$), find two different subsets that have the same sum.

8. (10%) A *cut-set* is a (minimal) set of edges in a graph such that the removal of the set will increase the number of connected components in the remaining subgraph, whereas the removal of any proper subset of it will not.
   (a) (5%) Show that, in a graph, a cut-set and any spanning tree must have at least one edge in common.
   (b) (5%) Let T_1 and T_2 be two spanning trees of a connected graph G. Let a be an edge that is in T_1 but not in T_2. Prove that there is an edge b in T_2 but not in T_1 such that both $(T_1 - \{a\}) \cup \{b\}$ and $(T_2 - \{b\}) \cup \{a\}$ are spanning trees of G.

9. (5%) A circuit of a graph is a path in which the terminal vertex coincides with the initial vertex. A *Hamiltonian circuit* is a circuit that passes through each of the vertices in a graph exactly once. The *undirected complete graph* of n vertices, denote K_n, is a graph with n vertices in which there is an edge between each pair of distinct vertices.
   For $n \geq 3$, how many different Hamiltonian circuits are there in K_n?

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