國　立　清　華　大　學　命　題　紙

95 學年度 資訊系統與應用 系（所） 甲 組碩士班入學考試

科目 工程數學 科目代碼 2702 共 4 頁第 1 頁 *請在【答案卷卡】內作答

(25%) I. Answer the following questions.

1. (5%) Find an upper triangular matrix A that satisfies $A^3=\begin{bmatrix}1 & 30\\ 0 & -8\end{bmatrix}$.

2. (5%) Let $B=\begin{bmatrix}1 & 3 & 7 & 11\\ 0 & 1/2 & 3 & 8\\ 0 & 0 & 0 & 4\\ 0 & 0 & 0 & 2\end{bmatrix}$. Find the eigenvalues of B^9.

3. (5%) Let R^4 have the Euclidean inner product. Express $\boldsymbol{w}=[-1, 2, 6, 0]^t$ in the form $\boldsymbol{w}=\boldsymbol{w}_1+\boldsymbol{w}_2$, where $\boldsymbol{w}_1$ is in the space W spanned by $\boldsymbol{u}_1=[-1, 0, 1, 2]^t$ and $\boldsymbol{u}_2=[0, 1, 0, 1]^t$, and $\boldsymbol{w}_2$ is orthogonal to W.

4.(10%) Find det(A) given that A has $p(\lambda)$ as its characteristic polynomial.

(a) $p(\lambda)= \lambda^3-2\lambda^2+\lambda+5$　　(b) $p(\lambda)= \lambda^4-\lambda^3+7$

(25%) II. Answer the following questions.

1. (5%) Find a, b, and c such that the matrix

$$A=\begin{bmatrix} a & 1/\sqrt{2} & -1/\sqrt{2} \\ b & 1/\sqrt{6} & 1/\sqrt{6} \\ c & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}$$

is orthogonal.

2. (5%) Consider the basis $S=\{ v_1, v_2, v_3\}$ for R^3, where $v_1=[1, 2, 1]^t$, $v_2=[2, 9, 0]^t$, and $v_3=[3, 3, 4]^t$, and let T: $R^3 \rightarrow R^2$ be the linear transformation such that

$$T(v_1)=[1, 0]^t, T(v_2)=[-1, 1]^t, \text{ and } T(v_3)=[0, 1]^t.$$

Let $w=[7, 13, 7]^t$, find $T(w)$.

3. (5%) Let $a_1=[1, 1, 0]^t$, $a_2=[2, 3, 0]^t$, and $b=[4, 5, 6]^t$. Find the projection vector of b onto the plane that is spanned by the vectors a_1 and a_2.

4. (10%) The owner of a rapidly expanding business finds that for the first five months of the year the sales (in thousands) are \$4.0, \$4.4, \$5.2, \$6.4, and \$8.0. The owner plots these figures on a graph and conjectures that for the rest of the year the sales curve can be approximated by a quadratic polynomial. Find the least squares quadratic polynomial fit to the sales curve, and use it to project the sales for the twelfth month of the year.

(25%) III. Answer the following questions.

1. Let X and Y have the joint p.d.f. $f(x, y) = e^{-2}/[x!(y-x)!]$, $y = 0, 1, \ldots$; $x = 0, 1, \ldots, y$, zero elsewhere.
   (a)(5%) Find the moment-generating function $M(t_1, t_2)$ of this joint distribution.
   (b)(5%) Compute the means, the variances, and the correlation coefficient of X and Y.
   (c)(4%) Determine the condition mean $E(X|y)$.

1. (6%) Suppose that a woman leaves for work between 8:00 a.m. and 8:30 a.m. and takes between 40 and 50 minutes to get to the office. Let X denote the time of departure and let Y denote the time of travel. If we assume that these random variables are stochastically independent and uniformly distributed, find the probability that she arrives at the office before 9:00 a.m.

2. (5%) Let X have the distribution $F(x)$ of the continuous type that is strictly increasing on the support $a < x < b$. Prove that the random variable Y, define by $Y = F(X)$, has a distribution that is U(0, 1).

(25%) IV. Answer the following questions.

1. (8%) Let $\Gamma(x) = \int_0^\infty e^{-t}t^{x-1}dt$, $0 < x < \infty$ and let the r.v. X have the p.d.f.

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1, \quad where \ \alpha, \beta > 0$$

   (a) Show that $\Gamma(x+1) = x\Gamma(x)$, $0 < x < \infty$.

   (b) Find the expectation $E(X)$ and the variance $Var(X)$.

   [*Hint*] $\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$.

2. (8%) Let the random variable X have the moment generating function $M_X(t) = e^{-3t+2t^2}$.

   (a) Give the probability density function of X.

   (b) Define $Y = \frac{X+3}{2}$, what is the distribution of Y?

   (c) Define $Z = Y^2$, what is the distribution of Z?

3. (9%) Let $\{X_1, X_2, \cdots, X_n\}$ be a random sample of Poisson distribution with the mean λ.

   (a) Give the probability density function of X_1.

   (b) Give the moment generating function of X_1.

   (c) Define $Y = X_1 + X_1 + \cdots + X_n$, calculate the moment generating function of Y.