國 立 清 華 大 學 命 題 紙

九十二學年度 資訊系統與應用 系（所） 甲 組碩士班研究生招生考試

科目 線性代數 科號 2803 共 4 頁第 1 頁 *請在試卷【答案卷】內作答

(25%) I. Fill in the blanks.

1.

$$det\begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 1 & 1 & -2 & 3 \end{pmatrix} = \_\_\_\_\_\_\_\_$$

2. Let

$$A = \begin{pmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{pmatrix}. \qquad A^{-1} = \_\_\_\_\_\_\_\_$$

3. The angle between $x = [1,1,1,1]^T$ and $y = [8,2,2,0]^T$ is ________.

4. If A is an $m \times n$ matrix of rank r, then $dim(N(A^T)) =$ ________.

5. Let

$$A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{pmatrix}.$$

The dimension of the eigenspace corresponding to the eigenvalue $\lambda = 1$ is ________.

(25%) II. Fill in the blanks.

1. Let $V = span([1,0,1]^t) \subset R^3$, then $V^{\perp} =$ ________

2. Let $\mathbf{x} = [2,4,\ldots,2n]^t$, then $\|\mathbf{x}\|_1 =$ ________

3. Let $\mathbf{v} = [1,1,1]^t$ and $\mathbf{b} = [2,4,6]^t$, then the projection of $\mathbf{b}$ onto the line $\mathbf{v}$ is ________

4. Let

$$B = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

The eigenvalues of B^{-1} are ________

5. Let $L : R^3 \rightarrow R^3$ be a linear transform defined as $L([a,b,c]^t) = [0,0,b+c]^t$, then $dim(Ker(L)) =$ ________

(20%) III. Mark ○ if the statement is true, and mark × otherwise.

1. If S and T are subspaces of a vector space V, then $S \cup T$ is a subspace of V.
2. If $\mathbf{x}$ is a nonzero vector in R^n and $A\mathbf{x} = \mathbf{0}$, then $det(A) = 0$.
3. Let $E = [x_1, x_2, \ldots, x_n]$ be an ordered basis for R^n. If $L_1 : R^n \to R^n$ and $L_2 : R^n \to R^n$ have the same matrix representation with respect to E, then $L_1 = L_2$.
4. If U, V, and W are subspaces of R^3, then $U \perp V$ and $V \perp W$ imply $U \perp W$.
5. If A is an $n \times n$ matrix, then A and A^T have the same eigenvalues.
6. A basis of a vector space is an orthogonal set.
7. Let A be an $n \times n$ real matrix, then A has rank n^2.
8. An orthogonal set of vectors in a vector space are linearly independent.
9. Every diagonally dominant real matrix is positive definite.
10. Every real symmetric matrix can be diagonalized.

(30%) IV. Choose the best answer in the following questions.

1. α is a scalar and A, B, and C are matrices for which the indicated matrix operations are defined. Which on the following statements is false?
   (a) $A(B + C) = AB + AC$
   (b) $\alpha(AB) = A(\alpha B)$
   (c) $(A + B)^2 = A^2 + 2AB + B^2$
   (d) $(AB)^{-1} = B^{-1}A^{-1}$
   (e) all of the above
2. Which one of the following statement is not equivalent to the rest for an $n \times n$ matrix A?
   (a) A is nonsingular
   (b) $Ax = 0$ has only the trivial solution
   (c) A is row equivalent to the identity matrix
   (d) $det(A) \neq 0$

(e) The row vectors of A form a basis for R^n.

3. Given

$$A = \begin{pmatrix} 4 & 1 & 2 & 1 \\ 5 & 2 & 4 & 2 \\ 2 & 4 & 3 & 4 \\ 1 & 3 & 2 & 3 \end{pmatrix}$$

Which is $det(A)$?

(a) -4

(b) 0

(c) 8

(d) 16

(e) none of the above

4. Let P_n denote the set of polynomials of degree less than n. Which of the following is a subspace of P_4?

(a) The set of polynomials in P_4 of even degree

(b) The set of polynomials in P_4 of degree 3

(c) The set of polynomials in P_4 such that $P(0) = 0$

(d) The set of polynomials in P_4 having at least one real root

(e) none of the above

5. Which one of the following statement is false?

(a) $Ax = b$ is consistent if and only if B is in the column space of A.

(b) The rank of A plus the nullity of A equals m, where A is an $m \times n$ matrix.

(c) The dimension of the row space of A equals the dimension of the column space of A.

(d) The null space of A equals the orthogonal complement of $R(A^T)$.

(e) The intersection of two orthogonal subspaces is the zero vector.

6. Let $\mathbf{u}, \mathbf{v} \in R^n$ be orthonormal vectors, then $\|3\mathbf{u} + 4\mathbf{v}\|_2 =$?

(a) 5

(b) 7

(c) $5n$

(d) $7n$

(e) none of the above

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7. Let $Q \in R^{n \times n}$ be orthogonal, then $\|Q\|_2^2 =$?
   (a) 0
   (b) 1
   (c) 2
   (d) n
   (e) n^2

8. Let $A \in R^{n \times n}$ have eigenvalues $2, 4, \cdots, 2n$, then $tr(A) =$?
   (a) n
   (b) n^2
   (c) $n(n+1)$
   (d) $n(n-1)$
   (e) none of the above

9. Let $A \in R^{m \times n}$ have rank r, then $dim(Null(A)) + dim(R(A)) =$?
   (a) $m - r$
   (b) $n - r$
   (c) m
   (d) n
   (e) none of the above

10. Let $A \in R^{3 \times 3}$ have eigenvalues $\lambda(A) = \{1, 2, 5\}$, then $\lambda(A^{-1}) =$?
    (a) {0,1,4}
    (b) {-1,-2,-5}
    (c) {1,8,125}
    (d) {-1,-0.2,-0.5}
    (e) {1,0.2,0.5}