國　立　清　華　大　學　命　題　紙

九十二學年度 資訊系統與應用研究 系（所） 甲 組碩士班研究生招生考試

科目 離散結構 科號 2802 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. (36%) Answer the following questions. (You don't have to explain how you derive them.)
   (a) (4%) A binary tree has 100 leaves. How many internal nodes does this binary tree have?

   (b) (4%) Among the integers 1-300, how many of them are divisible by 3, but not by 5 nor by 7?

   (c) (4%)Please complete the following inequalities/equalities by filling the blanks with $\geq$, =, or $\leq$. (a) $|P \cup Q| \_\_\_ |P|+|Q|$ (b) $|P \cap Q| \_\_\_ \min(|P|,|Q|)$ (c) $|P \oplus Q| \_\_\_ |P|+|Q|-2|P \cap Q|$

   (d) $|P-Q| \_\_\_ |P|-|Q|$.

   (d) (4%) Simplify the following expression:

$$\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}+\cdots+\frac{1}{(3n-2)\cdot(3n+1)}.$$

   (e) (4 %) Among 11 senators, in how many ways we can select a committee of 5 members so that at least one of senator A and senator B will be included?

   (f) (4%) How many different ways can we place $2n+1$ indistinguishable balls in three distinct boxes so that any two boxes together will contain more balls than the other one?

   (g) (4%) Let A={a, b, c, d, e, f, g, h, i, j, k} be a set. Let $\pi_1 = \{\overline{abcd}, \overline{efg}, \overline{hi}, \overline{jk}\}$ and

   $\pi_2 = \{\overline{abch}, \overline{di}, \overline{efjk}, \overline{g}\}$ be two partitions of A. (a) What is $\pi_1 \cdot \pi_2$? (b) What is $\pi_1 + \pi_2$?

   (h) (4%) If the first 10 positive integers are placed around a circle, in any order, there must exist three integers in consecutive locations around the circle that have a sum greater than or equal to A. What is the maximum value of A?

   (i) (4%) Consider the problem of connecting 19 lamps to a single electric outlet by using extension cords each of which has four outlets. How many extension cords are needed?

2. (12%) Briefly answer the following questions. (You don't have to give the proof.)
   (a) (4%) What is the sufficient and necessary condition for an undirected graph to possess a eulerian path?

(b) (4%) What is the definition of a hamiltonian path?

(c) (4%) What is the definition of a tree?

3. (4%) Professor Lai has just returned from a visit to an island where each inhabitant either always tells the truth or always lies. He told us that he heard the following statements made by two of the island's inhabitants A and B:

A: B always lies

B: A always tells the truth

What can you say about Professor Lai's vacation?

4. (20%) Briefly answer the following questions.

(a) (4%) If G is a graph with v vertices and e edges, how many edges can be removed without causing the remaining graph to be disconnected.

(b) (4%) The recursive definition of Ackermann's function is denoted by Ack: $N \times N \rightarrow N$, where N is the set of nonnegative integers,

Ack $(0, n) = n + 1$

Ack $(m, 0)$ = Ack $(m\text{-}1, 1)$ if $m > 0$

Ack (m, n) = Ack $(m\text{-}1,$ Ack $(m, n\text{-}1))$ if $n, m > 0$.

Compute Ack (3,2).

(c) (6%)Which of the following codes are prefix codes?

a. 101, 1101, 0101, 001, 1001, 11101, 0110, 01001
b. 1101,0011,1010,1110,0101,1100,1111
c. 101,1100,01010,10101,0011,0110
d. 1011,10101,10110,0101,01100,111000
e. 1101,10001,1001,01110,10110,110110

(d) (6%) Find a deterministic automata which accepts the language expressed by (a*(ba)*bb*a)*.

5. (4%)

(a) Give the definition of the time complexity of a problem.

(b) Give a possible way of locating the time complexity of a problem.

6. (6%) Consider the following instructions given to a maintenance technician:
   (i) The electric power should be turned on if it is not the case that nobody is in the office and automatic monitoring system is not in operation.
   (ii) The automatic monitoring system will be in operation if and only if nobody is in the office or a large payroll is left in the office.
   Show that the instructions can be replaced by the simple instruction of always leaving the electric power on.

7. (6%) The LARGESMALL problem is to determine both the largest and smallest of n given numbers. Design an optimal algorithm using $(3n/2) - 2$ operations to solve the LARGESMALL problem.

8. (6%) Use the generating function to prove that the number of partitions of the integer n into odd integers with repetitions allowed equals to the number of partitions of the integer n into distinct parts.

9. (6%) In a sequence of $n^2 + 1$ distinct integers, there is either an increasing subsequence of length $n + 1$ or a decreasing subsequence of length $n + 1$. (Hint: use the *pigeonhole principle*)