1. (10%) Let T be a linear transformation from R^2 to R^3 such that $T(1,1)=(1,2,3)$ and $T(3,2)=(3,2,1)$. What is $T(5,4)$?
2. (10%) What is the projection of (3, 4, 5) onto the plane spanned by (1, 1, 0) and (1, −1, 2)?
3. (10%) Find the matrix A whose eigenvalues are 1, 2, and 3 and whose eigenvectors are $\begin{bmatrix}2\\2\\1\end{bmatrix}$, $\begin{bmatrix}1\\6\\2\end{bmatrix}$, and $\begin{bmatrix}3\\1\\1\end{bmatrix}$, respectively.
4. (10%) Assume A and B are two square matrix of the same dimension. Prove that AB and BA have the same set of eignevalues.
5. (10%) Assume A is a square matrix. Prove that 1 is an eigenvalue of A if the sum of the entries in each row of A is 1.
6. (10%) Prove that if A is positive definite, then the eigenvalues of A are positive.
7. (10%) Compute the eigenvalues and corresponding eigenvectors of matrix $A=\begin{bmatrix}3 & -1 & 1\\-2 & 2 & -1\\-2 & 0 & 1\end{bmatrix}$. (Please note that all elements in the obtained eigenvectors should be expressed in integers.)
8. (15%) Solve the matrix equation $A^2-3A+I=\begin{bmatrix}-7 & 6\\-12 & 11\end{bmatrix}$, where I is an identity matrix.
9. (15%) Prove that for a real symmetric matrix, all eigenvalues are real and eigenvectors corresponding to distinct eigenvalues are orthogonal.