國 立 清 華 大 學 命 題 紙

97 學年度 資訊工程學 系（所） 組碩士班入學考試

科目 基礎計算機科學 科目代碼 2001 共 3 頁第 1 頁 *請在試卷【答案卷卡】內作答

1. (6%) How many ways are there to choose 10 items from 6 distinct items when
   (a) (3%) the items in the choices are ordered and repetition is allowed?
   (b) (3%) the items in the choices are unordered and repetition is allowed?

2. (3%) Let $L(x, y)$ be the statement "x likes y," where the universe of discourse for both x and y consists of all people in the world. Use quantifiers to express the statement "There is someone who likes no one besides himself or herself."

3. (10%) Suppose that a valid codeword is an n-digit number in decimal notation containing an even number of 0s. Let a_n denote the number of valid codewords of length n.
   (a) (5%) Give a recurrence relation for the sequence $\{a_n\}$.
   (b) (5%) Use generating functions to find an explicit formula for a_n.

4. (6%) Let S be the set of all strings of English letters. Consider the following relations on S and determine whether
   (a) (2%) $R_1 = \{(a, b) | a \text{ and } b \text{ have no letters in common}\}$ is reflexive or not.
   (b) (2%) $R_2 = \{(a, b) | a \text{ and } b \text{ are not of the same length}\}$ is transitive or not.
   (c) (2%) $R_3 = \{(a, b) | a \text{ is longer than } b\}$ is symmetric or not.
   Justify your answers.

5. (4%) Read the following sentences and answer the questions. Note that explanation is necessary.
   (a) It takes exponential number of steps to solve any NP-complete problem in the worst case.
   (b) The time complexity of any algorithm that solves a problem is an upper bound on the time complexity of this problem.

6. (5%) There is a barber in a small village. He claims that he will shave everybody who does not shave himself. Show that there is no such barber can exist.

7. (5%) Show that every planar graph has a node of degree at most 5.

8. (6%) Entropy $H(X)$ is a measure of the average uncertainty associated with a random variable X, usually defined as

$$H(X) = -\sum_{\forall x} P_X(x) \log_2 P_X(x).$$

   Conditional entropy $H(X|Y)$ is a measure of the average uncertainty in Y when the realization of X is observed.
   (a) (4%) Prove the chain rule, $H(X, Y) = H(X) + H(X|Y)$.
   (b) (2%) Show that $H(Y) - H(Y|X) = H(X) - H(X|Y)$.

9. (5%) Given a weighted graph $G = \{V, E, w\}$ and two distinct vertices a and b, where w is a function from E to the set of positive real numbers, what is the time complexity of the following algorithm on G?

   (1) Initially, let $P = \{a\}$ and $T = V-\{a\}$. For every vertex t in T, let $l(t) = w(a, t)$.

   (2) Select the vertex in T that has the smallest index with respect to P. Let x denote such vertex.

   (3) If $x = b$, stop. If not, let $P' = P \cup \{x\}$ and $T' = T- \{x\}$. For every vertex t in T', update its index with respect to P'.

   (4) Repeat Steps (2) and (3) using P' as P and T' as T.

10. (6%) A typical node in a doubly linked list has three fields: a data field, a left link field, and a right link field. To improve space efficiency, we may implement a doubly linked list using only two fields per node: a data field and a link field that is derived from the left and right links using the bitwise XOR operation.

    Suppose the list `{HAT, CAT, EAT, BAT}` is stored as a doubly linked list by the following three-field representation:

Index	1	2	3	4
Data	HAT	CAT	EAT	BAT
Left link	0	1	2	3
Right link	2	3	4	0

    What are the values of (a)-(d) for the corresponding two-field representation?

Index	1	2	3	4
Data	HAT	CAT	EAT	BAT
Link	(a)(1%)	(b)(1%)	(c)(1%)	(d)(1%)

    (e) (2%) Explain how to traverse a two-field doubly linked list from left to right and from right to left.

11. (5%)

    (a) (2%) The failure function of the Knuth-Morris-Pratt algorithm for the pattern `abcabcacab` can be expressed as

a	b	c	a	b	c	a	c	a	b
-1	-1	-1	0	1	2	3	-1	0	1

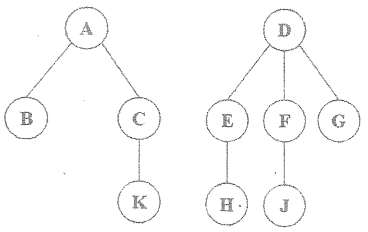
    What is the failure function for the pattern `aabaabaaab`?

    (b) (3%) Briefly describe how to use the Knuth-Morris-Pratt algorithm to determine if a string S is a cyclic rotation of another string S' in linear time. For example, `tea` and `eat` are cyclic rotations of each other.

12. (14%)

(a) (2%) For any nonempty binary tree T, if n is the number of leaf nodes and m the number of nodes of degree 2. What is the relation between n and m?

(b) (3%) Transform the following two-tree forest into the corresponding binary tree.



(Problem 12(b))

(c) (3%) Find all binary trees whose nodes appear in exactly the same sequence in both preorder and inorder.

(d) (3%) Suppose we start with n sets, each containing a distinct element. If u unions are performed, what is the smallest number of remaining singleton sets?

(e) (3%) Draw the **threaded binary tree** of nine nodes that are *A B D C E G F H J* in preorder and *D B A E G C H F J* in inorder.

13. (12%) Given a graph $G =$

	0	1	2	3	4	5	6
0	0	28	0	0	0	10	7
1	28	0	16	0	0	0	14
2	0	16	0	12	0	0	0
3	0	0	12	0	22	0	18
4	0	0	0	22	0	25	24
5	10	0	0	0	25	0	0
6	7	14	0	18	24	0	0

, where nonzero entry represents the weight of an edge of G.

(a) (3%) Find all the articulation points of graph G.

(b) (3%) Find all the bi-connected components of graph G.

(c) (3%) The BFS sequence of G (assume that the search always starts from the node with the smallest rank).

(d) (3%) The DFS sequence of G (assume that the search always starts from the node with the smallest rank).

14. (5%) Given an input sequence as Aug, Dec, July, Feb, May, Mar, Oct, Apr, Jan, Jun, Sep, Nov. What is the max heap structure of the given sequence assume that the heap is empty initially?

15. (8%) Let $n = 5$ and $(a_1, a_2, a_3, a_4, a_5)$ = (do, for, if, return, while) be an identifier set. Let $(p_1, p_2, p_3, p_4, p_5)$ = (1, 1, 3, 3, 2) and $(q_0, q_1, q_2, q_3, q_4, q_5)$ = (2, 3, 1, 2, 1, 1) be the probabilities for the successful and unsuccessful search of identifiers, respectively. Note that the p's and q's have been multiplied by 20 for convenience. What is the cost of the optimal binary search tree for (do, for, if, return, while)?