

國立清華大學命題紙

96 學年度 資訊工程學系 (所) _____ 組碩士班入學考試

科目 基礎計算機科學 科目代碼 2101 共 3 頁第 1 頁 *請在【答案卷卡】內作答

1. (7%) Compute the failure function f of the following string:

$P[1..8] = \text{"ABABABCA"}$

2. (8%) Insertion sort is efficient for sorting a small size of data items. We can modify mergesort as follows: When mergesort is called on a subarray with fewer than k elements, where k is an integer $\leq n$, we stop recursive call and run insertion sort on the subarray. What's the worst-case time complexity of the above sorting algorithm, in terms of n and k ? Justify your answer.

3. (10%) Let $S = \{s_1, s_2, \dots, s_n\}$ be a set of n positive integers and $t > 0$ be an integer. The subset-sum problem is to find a subset of S that adds up exactly to t . Suppose that $t = O(n^{3.5})$. How fast can we solve the subset-sum problem? Describe your algorithm in detail.

4. (7%) The LARGESMALL problem is to determine both the largest and smallest of n given numbers. Design an optimal algorithm to solve the LARGESMALL problem with $(3n/2) - 2$ operations. Justify your answer.

5. (10%) Read the following sentences and answer the questions. Note that explanation is necessary.

- (a) (5%) The following program is a count-controlled loop going from 1 to 5. At each iteration, the loop counter is either printed or put on a stack depending on the result of Boolean function `RandomFunc()` (Here the behavior of `RandomFunc()` is immaterial). At the end of the loop, the items on the stack are popped and printed. Due to the logical properties of a stack, this program cannot print certain sequences of the values of the loop counter. You will be given an output and asked to determine if the program could generate the output.

```
for (count = 1; count <= 5; count++)
    if (RandomFunc())
        cout << count;
    else
        stack.Push(count);
while (!stack.IsEmpty())
{
    stack.Pop(number);
    cout << number;
}
```

The following output is possible using a stack: 1 3 5 2 4

(1) True (2) False (3) Not enough information

Your answer is _____.

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科目 基礎計算機科學 科目代碼 2101 共 3 頁第 2 頁 *請在【答案卷卡】內作答

(b) (5%) Again, the following program is a count-controlled loop going from 1 to 5. At each iteration, the loop counter is either printed or put on a queue depending on the result of Boolean function `RandomFunc()` (Here the behavior of `RandomFunc()` is immaterial). At the end of the loop, the items on the queue are dequeued and printed. Due to the logical properties of a queue, this program cannot print certain sequences of the values of the loop counter. You are given an output and asked to determine if the program could generate the output.

```
for (count = 1; count <= 5; count++)
    if (RandomFunc())
        cout << count;
    else
        queue.Enqueue(count);
while (!queue.IsEmpty())
{
    queue.Dequeue(number);
    cout << number;
}
```

The following output is possible using a queue: 1 3 5 4 2

(1) True (2) False (3) Not enough information

Your answer is _____.

6. (15%) Evaluate the following five questions to either **True** or **False**. You have to briefly explain your answer. Same answer to all questions or no explanation will be treated as zero mark directly.

- A high-probability ordering scheme would be a poor choice for arranging an array of elements that are equally likely to be requested.
- When a hash function is used to determine the placement of elements in an array, the order in which the elements are added does not affect the resulting array.
- A sequential list is always stored in a statically allocated structure.
- Recursive functions generally use `While` or `For` statements as their main control structure.
- Recursive functions are always less "efficient," in terms of Big-O.

7. (18%) Evaluate the following nine questions to either **True** or **False**.

You don't have to explain the reasoning.

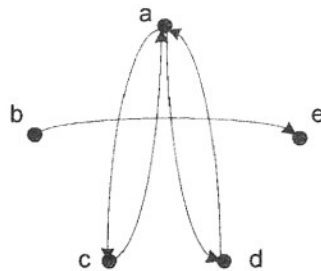
- An undirected graph possesses an eulerian path if and only if it is connected and has either zero or two vertices of even degree.
- It is possible to write a program that will examine any student's program together with the data it works on and report whether the program, working on the given data, will ever stop.

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- (c) A problem is considered tractable if it can be solved by an $O(n^k)$ -time algorithm, where k is a constant.
- (d) If a statement tells us the occurrence of a certain event that is not likely to happen, we would say that the statement contains a small amount of information.
- (e) If factoring a large integer is easy, then breaking the RSA public key cryptosystem is also easy.
- (f) In a sequence of $n^2 + 1$ distinct integers, there is either an increasing subsequence of length $n + 1$ or a decreasing subsequence of length $n + 1$.
- (g) A subset of A is called a *chain* if every two elements in the subset are related (comparable). A subset of A is called an *antichain* if no two distinct elements in the subset are related. Let (P, \leq) be a partially ordered set. Suppose the length of the longest chains in P is n . Then the elements in P can be partitioned into n disjoint antichains.
- (h) Two states are k -equivalent if and only if they are 1-equivalent and, for any input letter, their successors are $(k-1)$ -equivalent.
- (i) The language $L = \{xx \mid x \text{ is a string of 0s and 1s}\}$ is a finite state language.
8. (6%) Let $B = \{\phi\}$, C is the power set of B , i.e., $C = P\{B\}$ and $D = P\{C\}$.
- (a) (2%) $C - D = ?$
- (b) (2%) $P(\{a, \{\phi\}\}) \oplus D = ?$
- (c) (2%) Is $\{\{\phi\}\} \in D$? Is $\{\{\phi\}\} \subseteq D$?
9. (5%) Three distinct students got into the elevator on the ground floor of our 8-floor EECS building. What is the probability that they will all get off at different floors?
10. (5%) Cynthia has 9 classmates in a computer programming class. For the final project as teamwork, in how many ways can she team up with three or more of them?
11. (9%) Given a relation R with the directed graph as shown below:



- (a) (2%) Find the reflexive closure of R with the directed graph.
- (b) (4%) Find the transitive closure of R with the directed graph.
- (c) (3%) Is the transitive closure of R an equivalence relation? State your reason.