

國 立 清 華 大 學 命 題 紙

九十三學年度 資訊工程學 系(所) _____ 組碩士班入學考試

科目 基礎計算機科學 科號 3001 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. When we do tree search for a set of very long keys, the cost of a key comparison is very high. How much can we reduce the number of key comparison by using a digital search tree such as a Patricia tree (Practical algorithm to retrieve information in alphanumeric)? (5%)
2. Describe how to build a Patricia tree of a set of keys, $K = \{0000, 0001, 0010, 1000, 1001, 1100\}$ in three steps:
 - (A) Build a binary tries for K with branch and element nodes (3%)
 - (B) Build a binary tries for K with degree one node removed (3%)
 - (C) Compress the above and build a Patricia tree for K (4%)
3. (A) Describe briefly a fast algorithm for UNION and FIND operations for a collection of sets of keys. (5%)
(B) Describe two cases where UNION and FIND are used as the basis operations for a more complicated problem. (5%)
4. Please give one fastest algorithm for finding the Minimum Cost Spanning Tree of a graph. (10%)
5. (A) How fast can we sort n numbers if all numbers are positive (and less than n^5) integers? Please briefly explain how to do the sorting? (10%)
(B) We already know that the sorting problem has lower bound, please explain why the well-know sorting lower bound does not apply to this sorting problem. (5%)
6. Find the general solution of the recurrence relation $2a_n = 7a_{n-1} - 3a_{n-2} + 2^n$ by using the method of generating functions. Please give the detailed procedure. (7%)
7. Construct a finite-state machine that determines whether the word "begin" or "end" has been read as the last five or three characters, respectively, in the input read so far. Please give the state diagram of this finite-state machine. Note that the input can be any string of English letters and the set of all the 26 English letters is denoted by the symbol Σ . (10%)

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8. Let $(A, \vee, \wedge, \bar{\quad})$ be a Boolean algebra, where the symbols \vee, \wedge and $\bar{\quad}$ denote the join, meet and complementation operations, respectively. Two operations \oplus and \square for a, b in A are defined as follows:

$$a \oplus b = (a \wedge \bar{b}) \vee (\bar{a} \wedge b),$$

$$a \square b = (\bar{a} \vee b) \vee (\bar{a} \wedge b).$$

- Are these two operations commutative? Explain your answers in details. (8%)

9. For $A = \{a, \{a\}, \{a, b\}, \Phi\}$, determine the following sets:

- (a) $A - \{a\}$
- (b) $\{a, b, c\} - A$
- (c) $(A \cup \{a, b\}) \cap \{\Phi\}$ (6%)

10. Among the integers 1 to 300, how many of them are not divisible by 3, or by 5, or by 7? (5%)

11.

- (a) Suppose that repetitions are not permitted. How many four-digit numbers can be formed from the six digits 1, 3, 5, 6, 8, 9?
- (b) How many of the numbers in part (a) are less than 5000?
- (c) How many of the numbers in part (a) contain both the digit 1 and the digit 9? (9%)

12. Let R be a binary relation on the set of all positive integers such that $R = \{(a, b) \mid a - b \text{ is an odd positive integer}\}$.

- (a) Is R reflexive?
- (b) Is R symmetric?
- (c) Is R anti-symmetric?
- (d) Is R transitive?
- (e) Is R an equivalence relation? (5%)