1. (15%) (a) (5%) Let A be a square matrix. Show that A and A^T have the same eigenvalues.

   (b) (5%) Find the eigenvalues and the corresponding eigenvectors of the matrix A:

$$A=\begin{pmatrix}2 & -1\\ 4 & -3\end{pmatrix}.$$

   (c) (5%) Solve the initial value problem:

$$\begin{cases} y_1' = 2y_1 + 4y_2 \\ y_2' = -y_1 - 3y_2 \end{cases}, \quad y_1(0)=2, y_2(0)=1.$$

2. (8 %) A box contains four balls numbered 1 through 4. The balls are selected one at a time without replacement. A match occurs if the ball numbered k is the k-th ball selected. Let the event A_i denote a match on the i-th draw, i=1, 2, 3, 4.

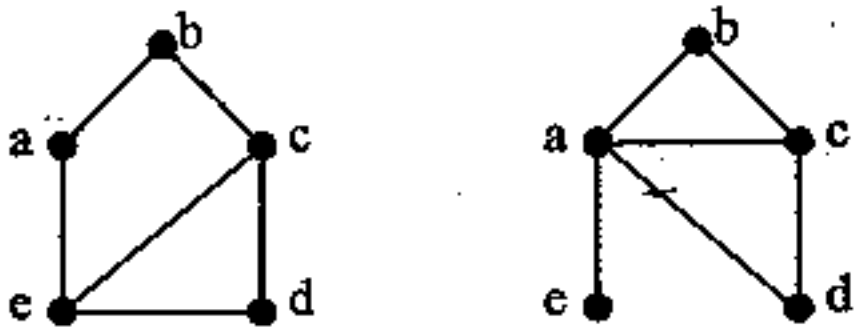
   (a) (3%) Find the probability $P(A_i \cap A_j)$ where $1 \le i < j \le 4$

   (b) (5%) Find the probability $P(A_1 \cup A_2 \cup A_3 \cup A_4)$

3. (7%) Give a tree representation and evaluate the value of the prefix expression

   + - * 2 3 5 / * 2 3 6

4. (5 %) Are the following two graphs isomorphic? Give reasons for your answer.



5. (10 %) Assume that there are n numbers in which every number is larger than 0 and less than 10000*n. Please design a fastest sorting algorithm to sort this n numbers. What is the time complexity of this fastest sorting algorithm in big-O notation?
6. (8%) How can you find the left and the right child of a node in an array (without left and right child links) representing a binary tree?
7. (8 %) Specify the operations step by step to delete a node (denoted by the variable x) in a doubly linked circular list? Suppose that each node x is represented by three arrays: DATA[x], LLINK[x] and RLINK[x]. (To simply your answer, you may assume that the node to be deleted is in the middle of the list and there are at least three nodes in the list .)

8. (10%) How fast can we find the minimum weighted spanning tree for a graph with n vertices and with each edge weight being n^2? What algorithm should we use?

9. (5%) Using the error-correcting code represented in Table 1.below to decode the following message: 001011 100100 001100

Table 1

symbol	code
A	000000
B	001111
C	010011
D	011100
E	100110
F	101001
G	110101
H	111010

10. (8 %) A floating-point number in terms of an 8-bit byte representation consists of 3 fields: a sign bit field, a 3-bit exponent field in excess-3 notation, and a 4-bit mantissa field. What real number does the floating-point number 10111010 represent?

11. (6%) In terms of the following relations X and Y in relation database, what is the appearance of the relation RESULT after executing the instruction:

RESULT ← JOIN X and Y where X.W ≧ Y.R

X relation

U	V	W
A	Z	5
B	D	3
C	Q	5

Y-relation

R	S
3	J
4	K

12. (5%) Suppose the following solutions have been proposed for removing the deadlock that occurs on a single-lane bridge when two cars meet. Identify which condition(s) for the deadlock given in the text is removed by each solution.

a. Do not let a car onto the bridge until the bridge is empty

b. If cars meet, make one of them back up.

c. Add a second lane to the bridge

13. (5%) Which of the following layers in the network communication TCP/IP

protocol divides an original message into packets of smaller size and attaches each packet with an address?: 1) Application Layer 2) Transport Layer 3) Network Layer 4) Link Layer 5) Physical Layer