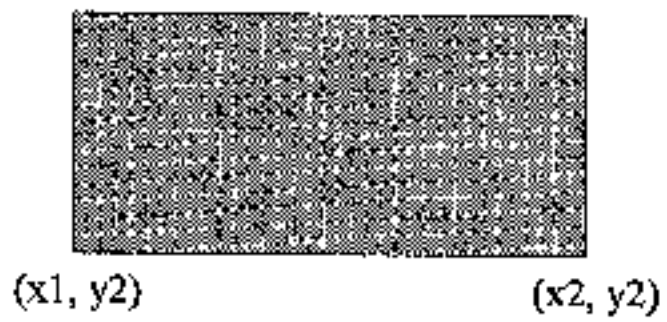
1. (10%) In this problem, we use two intervals [x1, x2] and [y1, y2] to represent the following rectangle:



Let P be the rectangle represented by [a, b] and [c, d]. And let Q be the rectangle represented by [e, f] and [g, h]. The area of P∩Q can be computed as follows:

Step 1: Determine the interval [w1, w2] which is the intersection of [a, b] and [e, f].
Step 2: Determine the interval [z1, z2] which is the intersection of [c, d] and [g, h].
Step 3: the area of P∩Q is (w2-w1)×(z2-z1).

Please follow the following instructions to implement Steps 1~3 as a computer subroutine. You can use either *Pascal* or *C*.

(a) (2%) Define **interv** as a data type consisting of two real numbers that represent an interval.
(b) (2%) Define **rectan** as a data type consisting of two **Interv** data that represents a rectangle.
(c) (3%) Write a function **I_Interv**. The function takes two formal parameters **I1** and **I2** of type **interv** as input. The output returned by the function is the intersection of **I1** and **I2**. (Return [0, 0] if the intersection is empty.)
(d) (3%)Write a function **I_Rectan**. The function takes two formal parameters **P** and **Q** of type **rectan** as input. The output of the function is the area of **P∩Q**.

2. (6%) Consider the following recursive function.

```
function maze(a, b, c: integer):integer;
begin
    if a<b then maze := a;                    /* return a */
    else maze := c * maze(a div b, b, c) + (a mod b);
                                              /* return c * maze(a div b, b, c) + (a mod b) */
end;
```

(Note that ***div*** denotes integer division and ***mod*** denotes modulus.)

(a) (2%) What is the returned value of the function call maze(1020, 10, 7)?
(b) (2%) What is the returned value of the function call maze(352, 4, 11)?
(c) (2%) What is the returned value of the function call maze(932798, 2, 2)?

3. (16%) Prove or disprove the following statements.

(a) (4%) Let disks in a Hanoi tower be numbered in order of decreasing size from 0 to n-1. The following HANOI program moves the i^{th} disk 2^i times.

```
HANOI(Start, Temp, End, n)
    if n = 1
        then
            move Start's top disk to End
        else
            HANOI(Start, End, Temp, n - 1)
            move Start's top disk to End
            HANOI (Temp, Start, End, n - 1)
```

(b) (4%) There is a circuit that can sort n numbers of 1-bit word by using only O(log n)-bit working space.

(c) (4%) Let T be a binary search tree, x be a leaf in T, and y be x's parent. The key of y is the smallest one in T which is larger than the key of x.

(d) (4%) Every binary tree is uniquely defined by its preorder and inorder sequences.

4. (12%) A procedure AAA is given in below:

```
AAA(A, p, r)
1.      x ← A[p]
2.      i ← p - 1
3.      j ← r - 1
4.      while TRUE
5.          do  repeat  j ← j - 1
6.              until   A[j] ≤ x
7.              repeat  i ← i + 1
8.              until   A[i] ≥ x
9.          if  i < j
10.             then    exchange A[i] ↔ A[j]
11.             else    return j
```

(a) (8%) Illustrate the operation of AAA on the array A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21], where p = 1 and r = 12, and guess the purpose of this procedure.

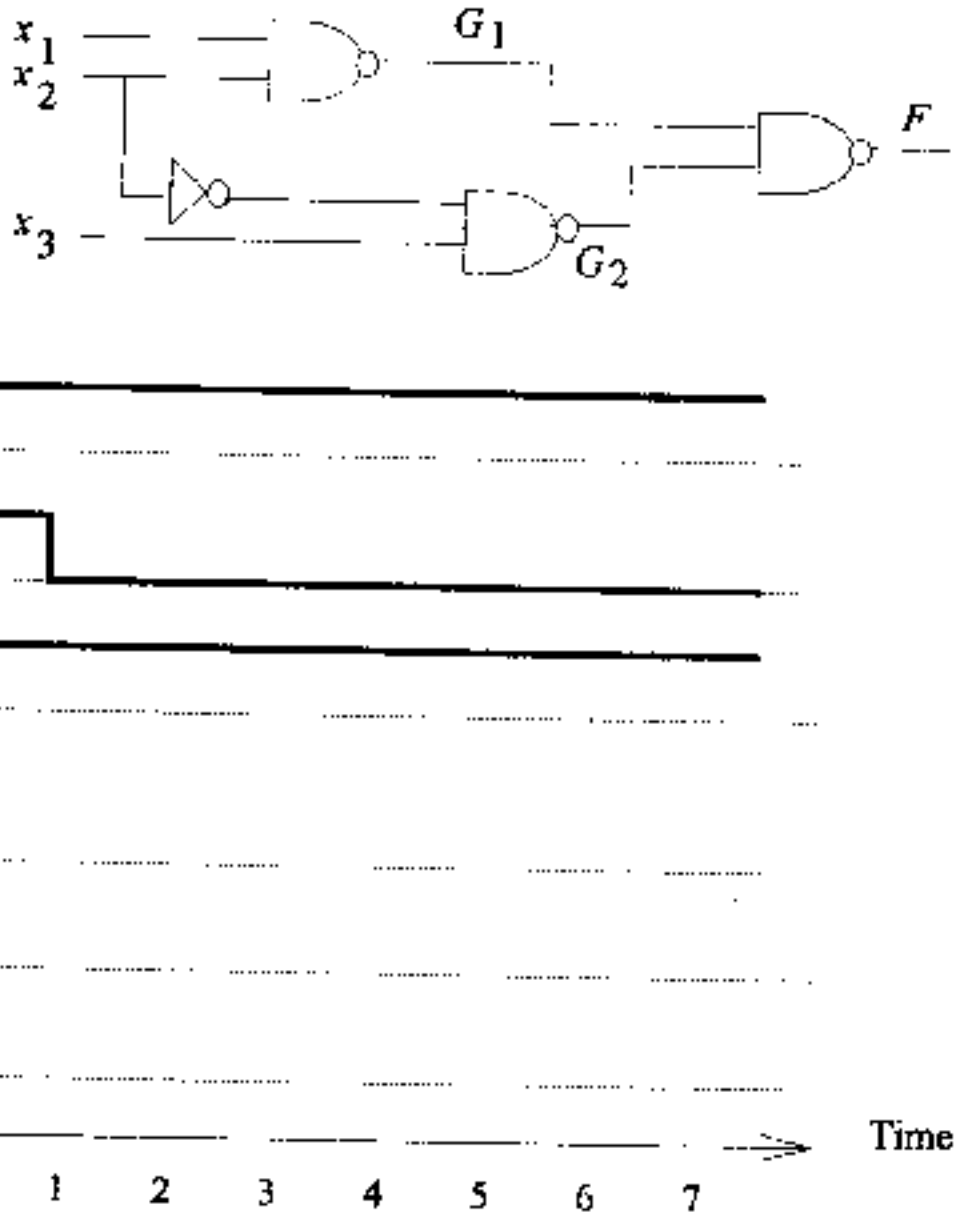
(b) (4%) Give a brief argument that the running time of AAA on an array of size n is $\Theta(n)$.

5. (6%) Compare the advantages and disadvantages of the following two string representation methods (Fixed Length and Linked List) for the following string operations.

Method	Concatenation	Pattern Matching	Substring Operations	Insertions and Deletions	Memory Utilization	Assignment
Fixed Length						
Linked List						

6. (a) (5%) Let the propagation delay of a NAND gate and an inverter gate be 2 and 1, respectively. Complete the timing diagram for the following circuit.

(b) (2%) What phenomenon do you observe at the output signal F?



7. (5%) Draw a logic diagram using only two-input NOR gates to implement the following function:

F(A, B, C, D) = $(AB+A'B')(CD'+C'D)$

8. (5%) Show how to realize the following functions using ROM.

$F_1(A, B, C) = A'B'+AC'$

$F_2(A, B, C) = AC'+B$

$F_3(A, B, C) = B+AC$

9. (10%) Explain the following terms.

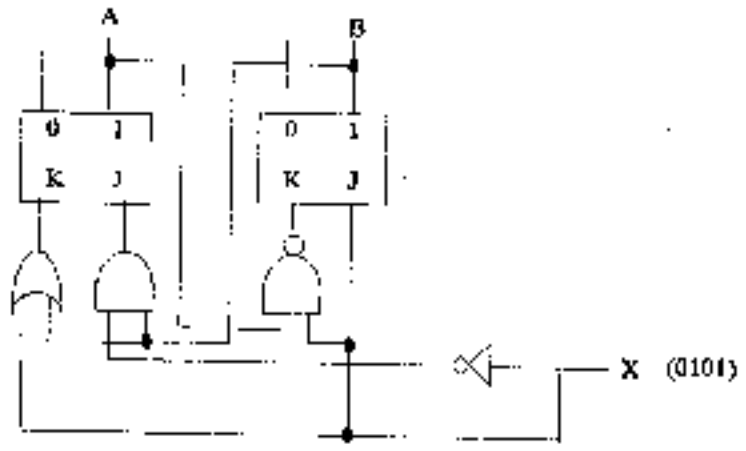
(a) WWW
(b) multiplexer
(c) call-by-value
(d) virtual reality
(e) EPROM

10. (5%) A four-input gate G realizes $G(A, B, C, D) = BC(A+D)$. Show the function

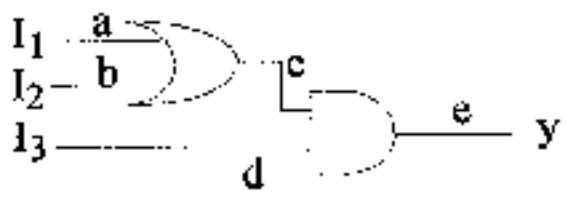
$$f(A, B, C, D) = \sum (0, 1, 2, 9, 10, 11, 13, 15)$$

can be realized by three G gates and one OR gate. (Assume both normal and complement inputs are available.)

11. (8%) What are the output sequences from J-K flip-flops A and B respectively if the input sequence of X is 0101 in the following sequential circuit? (Let the initial states of A and B be 0's.)



12. (5%) A single "stuck-at-x" fault can exist at an input or output terminal of a gate if it stays at x (x can be 0 or 1) under application of any input pattern. For the two-gate circuit below, find all the tests (input patterns) to detect existence of stuck-at-1 fault at terminal c.



13. (5%) Assign numbers from the set {0, 1, 2, 3, 4} to letters in the set {A, E, S, T, Y}. Your assignment must satisfy the arithmetical addition shown below: (Letters should be assigned by different numbers.)

$$\begin{array}{r} \text{S E E} \\ +\ \text{E A T} \\ \hline \text{Y E S} \end{array}$$