八十四學年度 資訊科學研究所 組碩士班研究生入學考試

科目 計算機導論 科號 0802 共 五 頁第 一 頁 *請在試卷【答案卷】內作答

1. (10%) Let m and n be two positive integers. Consider the following function MAZE.

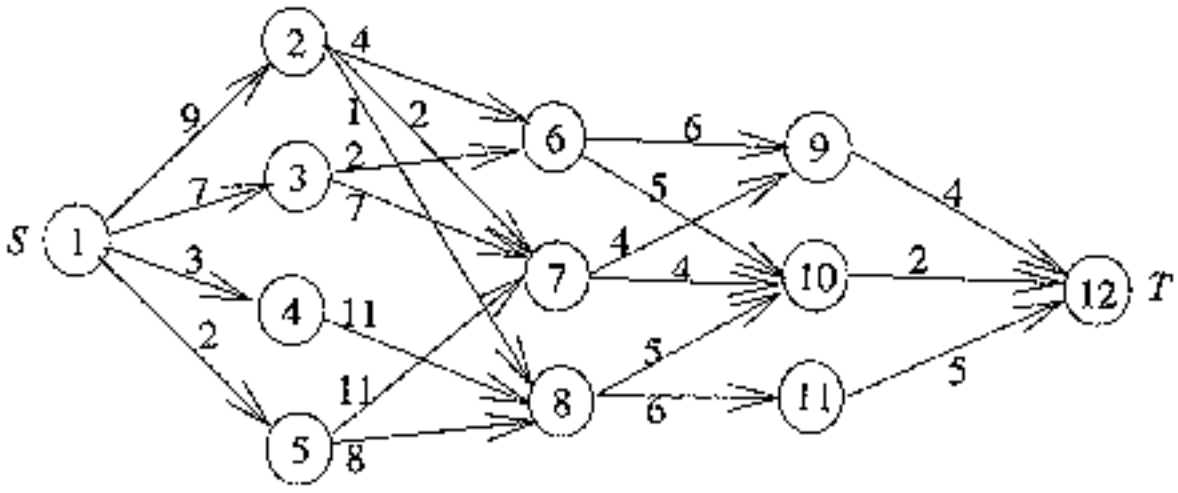
```
procedure MAZE(m, n:integer);
var
 temp, result: integer;
begin
   temp:=m;
   result:=0;
   while n<>0 do
   begin
      if (n mod 2)=1 then result:=result+temp;
      temp:=temp+temp;
      n:=n div 2;
   end;
   writeln(result);
end;
```

(1) What will be outputted if we perform the following procedure call MAZE(12, 11)? (3%)

(2) What's the purpose of the above procedure? (3%)

(3) What's the time complexity (in big-O notation with respective to m and n) of the above procedure? (4%)

2. (5%) This problem is to test your understanding of dynamic programming. Consider the following multistage graph. Show how to find the shortest path from the node S to the node T by dynamic programming (step by step).



3. (10%) Consider the following recursive procedure RECUR.

```
procedure RECUR(n:integer);
begin
   if n<=1 then write(n:4)
   else begin
      RECUR(n div. 2);
      write(n:4);
      RECUR(n div. 3);
   end;
end;
```

(1) What will be outputted if we perform the following procedure call RECUR(13)? (6%)

(2) What's the time complexity (in big-O notation with respective to *n*) of the above procedure? (4%)

4. (6%) Please answer the following questions briefly:

(a) We can use NAND gates only to implement all Boolean functions. Why? (2%)

(b) Is 'don't care' helpful in a K-map? Why? (2%)

(c) What is the DeMorgan's rule? Please give an example to show how DeMorgan's rule works. (2%)

5. (6%) What is the difference between:

   (a) PLAs (programmable logic arrays) and PAL (Programmable array logic). (2%)
   (b) One's complement and Two's complement. (2%)
   (c) DRAM (dynamic random-access memory) and SRAM (static random-access memory). (2%)

6. (10%) Please use minimum number of NAND gates (only) to implement an XOR gate. (4%) Please prove that the number of gates in your circuit is minimum. (6%)

7. (10%) Show how to implement a positive edge-triggered D flip-flop using two SR latches and some additional gates.

8. (10%) Assume each integer takes 1 unit of memory to store. Consider the array declaration

   Var A: array[1..5, 20..25, 10..15] of integer,

   and this array is stored in row major order.
   Answer the following questions:

   (1) How many spaces it takes to store array A? (2%)
   (2) If A[1, 20, 10] is stored at address 2000, what is the location of A[3, 20, 15]? (4%)
   (3) If A[1, 20, 10] is stored at address 2000, what is the array element at the location 2050? (4%)

9. (12%) Given a set T of n values, we define m to be the majority of T if and only if m is in T and the number of values, which are equal to m, is larger than $n/2$. For example, 4 is the majority of the set { 1, 4, 3, 4, 2, 4, 4, 4 }. And the majority of the set { 3, 4, 3, 4, 3, 4, 3, 4 } does not exist.

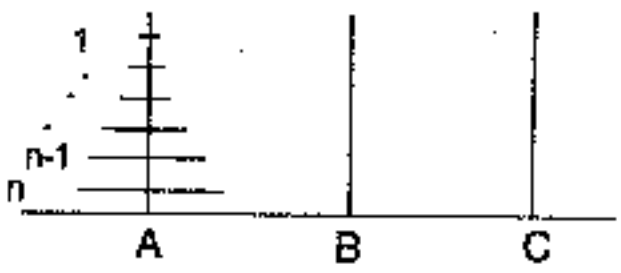
Given a set T of n values, we define k to be the medium of T if and only if k is in T and
the number of values, which are smaller than k, is less than $n/2$ and the number of values, which are smaller than or equal to k, is larger than or equal to $n/2$.

For example, 3 is the medium of the set { 3, 4, 3, 4, 3, 4, 3, 4 }.

(1) Prove the majority (if it exists) should be the medium. (5%)
(2) Briefly explain an algorithm which can find the majority in linear time. (3%)
(3) Briefly explain a data structure that stores values in linear spaces and supports the following two operations:
(a) Find the majority of the stored values in constant time.
(b) Insert an arbitrary value in $\log n$ time. (4%)

Notice that you should verify your designs to satisfy the time complexity requirements.

10. (15%) The Tower of Hanoi game is to place n circular rings of varying size 1 to n on three pegs A, B and C. Every step you are allowed to move one ring from one peg to the other and at any time, any ring of size i must not be placed on top of the other ring of size j if $i > j$. Initially, all rings are placed in peg A and the problem is to find the optimal number of steps that can move the n rings in peg A as shown in the following figure to peg C.



Tower of Hanoi game

(a) (3%) The optimal number of steps to move from peg A to peg C for n=3 is 7. What is the optimal number of moves for n = 6?

(b) (4%) If the optimal number of steps for moving n-1 rings from peg A to peg C is a_{n-1} what is the recurrent relation of a_n in terms of a_{n-1}?

(c) (4%) Prove that the a_n you find is indeed the optimal number of steps required to move n rings from peg A to peg C.

(d) (4%) What is the exact function for a_n in terms of n?

11. (6%) Suppose we remove a upper left corner square and a lower right corner square of a checkerboard of 6x6 squares to become a 6x6-2 checkerboard as shown in Fig. 11-a. Now we want to exactly cover this modified checkerboard with 17 dominoes of two adjacent squares as shown in Fig. 11-b. Can we do it or not? Explain why?

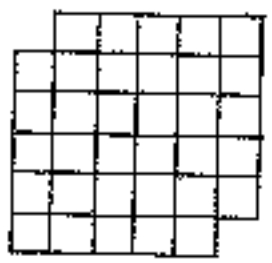


Fig. 11- a A 6x6-2 checkerboard

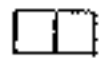


Fig. 11- b. A domino