國立清華大學 105 學年度碩士班入學考試試題

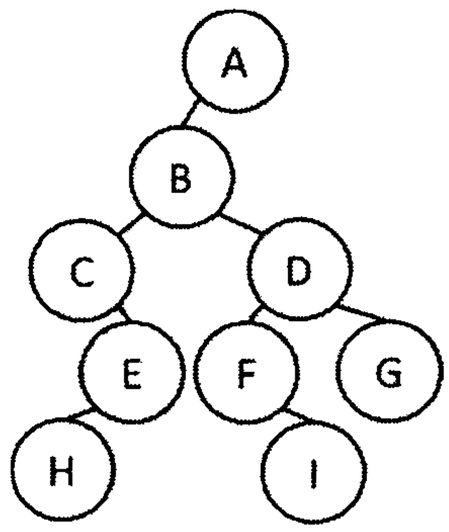
系所班組別：資訊工程學系

考試科目（代碼）：基礎計算機科學(1901)

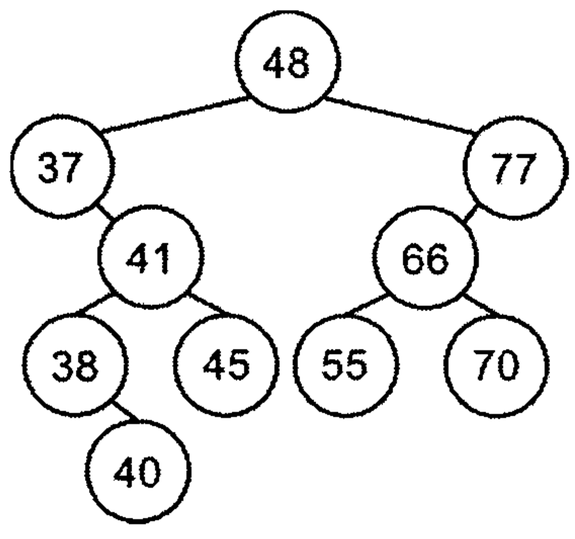
1. (17%) Answer each of the following questions.

(A) (2%) Given an infix expression (a+b)/c*(d-e), please write down the corresponding **prefix** and **postfix** expressions.

(B) (2%) Given the following binary tree, which corresponds to a **Left Child-Right Sibling** representation of a general tree. Please draw the input general tree.



(C) (2%) Given the following binary search tree (BST), please draw ALL possible tree(s) after deleting the node with key 48.



(D) (3%) Assume the **pre-order** and **in-order** traversal of a binary tree are ABDECFHGI and EDBAHFCGI, respectively. Please draw the tree, and explain whether or not the tree is unique.

(E) (3%) Assume the **pre-order** and **post-order** traversal of a binary tree are ABDECFHGI and EDBHFIGCA, respectively. Please draw the tree, and explain whether or not the tree is unique.

(F) (5%) A clocked tree is a binary tree in which each node n_i is associated with a non-negative delay, **delay(n_i)**. The path delay from a root to a node is defined as the summation of delay of all nodes along the path. The ***longestDelay*** is defined as the longest path delay among all root-to-leaf paths. Please base on the class definition show on the right to write pseudo codes of function:

void longestDelay(node *root, int AccDelay) ;

that computes and stores the longest path delay in the variable "MAX".

Class definition
class node{
int delay;
treenode *lchild;
treenode *rchild;
}
int MAX;

2. (17%) Answer each of the following questions.

(A) (3%)

Give a definition for the *reflexive transitive closure matrix* A^* of a directed graph G.

(B) (3%)

What does the following function compute?

```
void Function1(int cost[][MAX_VERTICES],
               int distance[][MAX_VERTICES], int n)
{
   /* cost is the adjacency matrix of some directed graph */
   int i, j, k;
   for (i=0; i<n; i++)
      for (j=0; j<n; j++)
         distance[i][j] = cost[i][j];

   for (k=0; k<n; k++)
      for (i=0; i<n; i++)
         for (j=0; j<n; j++)
            if (distance[i][k]+distance[k][j]<
                  distance[i][j])
               distance[i][j] =
                  distance[i][k] + distance[k][j];
}
```

(C) (3%)

Prove or disprove that a graph is bipartite only if it contains no cycles of odd length.

(D) (3%)

What is a topological sort (or topological ordering) of a directed acyclic graph?

(E) (5%)

Describe a linear time $O(|V|+|E|)$ algorithm for the topological sort of a graph $G=(V,E)$ using the depth first search implementation.

3. (17%) Answer each of the following questions.

(A) (5%) Let x and y be integers and m be a positive integer, then x is congruent to y modulo m if (x - y) is divisible by m, we denote this fact as $x \equiv y$ **mod** m. Fermat's Little Theorem states that if p is prime and z is an integer not divisible by p then $z^{p-1} \equiv 1$ **mod** p.

Use Fermat's Little Theorem to compute

3^{302} **mod** 5,

3^{302} **mod** 7,

3^{302} **mod** 11,

respectively.

(B) (5%) Use the results in (A) and Chinese Remainder Theorem to find 3^{302} **mod** 385. (Note that $385 = 5 \bullet 7 \bullet 11$)

(C) (7%) Solve the simultaneous recurrent relations

$a_n = 3a_{n-1} + 2\,b_{n-1}$

$b_n = a_{n-1} + \quad 2b_{n-1}$

with $a_0=2$, $b_0=1$

4. (17%)　Answer each of the following questions.

(A) (5%) How many different strings can be made by reordering the letters of the word *SUCCESS*?

(B) (7%) Assume that a sequence $\{a_n\}$ satisfies the recurrence relation

$$a_n = 8a_{n-1} + 10^{n-1}$$

and the initial condition $a_1 = 9$. Use generating functions to find an explicit formula for a_n.

(C) (5%) Consider the following relations on {1, 2, 3, 4},

- R_1={(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)},
- R_2={(1,1),(1,2),(2,1)},
- R_3={(1,1),(1,2),(1,4),(2,1),(2,2),(3,3),(4,1),(4,4)},
- R_4={(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)},
- R_5={(1,1),(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,3),(3,4),(4,4)},
- R_6={(3,4)}.

Which of these relations are symmetric ? anti-symmetric ? transitive?

5. (17%) Answer each of the following questions.

(A) (8%) Let u and v be two n bit numbers, where we assume that n is a power of 2 for simplicity. Clearly, multiplying u and v straightforwardly requires $O(n^2)$ steps.

(a) (4%) Please design a divide-and-conquer approach to compute the product of u and v in $O(n^{\log_2 3})$ time.

(b) (4%) Explain why the time complexity of your divide-and-conquer approach is $O(n^{\log_2 3})$.

(B) (9%) Determine whether the following statements are correct or not and also justify your answers. No points are given for answers without justification.

(a) (3 %) The Cook's theorem states that if the SAT (Satisfiability) problem is in NP, then P = NP.

(b) (3 %) If we want to prove that a decision problem X is NP-complete, it is enough to reduce X to the SAT (Satisfiability) problem.

(c) (3 %) For an NP-complete problem, we need to take exponential time to solve this problem for all kinds of inputs.

6. (15%) Answer each of the following questions.

(A) (10%) The following algorithm takes an n by n matrix A(1:n,1:n) as input, does LU-decomposition and stores the results in the same array A(1:n,1:n), where L is a unit lower triangular matrix (the diagonal elements are all 1s, and the elements above the diagonals are all 0s) and U is an upper triangular matrix (the elements below the diagonals are all 0s). We further assume that a given matrix can be LU-decomposed in our case.

```
for i=1,2,…,n-1
    for k=i+1,…,n
        t= - A(k,i)/A(i,i);
        for j=i+1,…,n
            A(k,j)=A(k,j)+t*A(i,j);
        endfor
        A(k,i)= -t;
    endfor
endfor
```

Given a 3 by 3 matrix as stated below as an input

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 4 & -1 \\ -1 & -3 & 3 \end{pmatrix}$$

What are the contents of A after implementing the algorithm? (Show integer parts only).

(B) (5%) Let $n \geq m$ be positive integers and let r be a nonnegative integers. The following algorithm will compute gcd(n,m), the greatest common divisor of n and m.

```
while (m>0) do
{r= n mod m;    n=m;    m=r;}
Return n
```

Given n=13923 and m=13056 as input, find the output of n.