國　立　清　華　大　學　命　題　紙

97 學年度 通訊工程研究 系（所） 乙 組碩士班入學考試

科目 基礎計算機科學 科目代碼 1902 共 2 頁第 1 頁 *請在【答案卷卡】內作答

1. Consider the positive integers between 1 and 100 (inclusive):
   (a) Find the number of integers that are not divisible by 2, 3, 5. $(2pts)$
   (b) How many integers must we choose in order to guarantee that in our random selection at least exist two integers x, y that $\gcd(x, y) \geq 2$? $(4pts)$

2. A standard deck have 13 spade cards, 13 heart cards, 13 diamond cards and 13 club cards. Suppose we shuffle the deck completely and label each card by giving a sequence number from 1 to 52.
   (a) How many possible arrangements that all spades are together? $(4pts)$
   (b) How many possible arrangements that no two spades are adjacent? $(4pts)$
   (c) How many possible arrangements that each spade is in front of all 13 hearts? $(6pts)$
   (i.e. the sequence number of each spade card must be smaller than the sequence number of every heart card)

3. Let $G = (V, E)$ be a complete simple graph where $|V| = n$ and $|E| = a_n$.
   (a) Derive a recurrence relation for a_n in terms of a_{n-1}. $(4pts)$
   (*Note* : you do not need to solve the recurrence relation)
   (b) Suppose $n = 6$, find the number of ***non-isomorphic*** spanning trees on G. $(6pts)$
   (c) Suppose $n = 6$, find the number of all possible spanning trees on G. $(6pts)$

4. Suppose that a tree has ten vertices of degree 2, ten vertices of degree 3, ten vertices of degree 4, one vertex of degree 5, and its remaining vertices have degree 1. How many vertices does the tree have? $(4pts)$

5. True or False . $(2pts \ldots each)$ (for each answer, you do not need to explain why.)
   (a) $(p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology.
   (b) Let $L(x, y)$ be the statement "x loves y", where the universe of discourse for both x and y consists of all people in the world. Then we can use " $\exists y, \forall x, L(x, y)$" to express the statement "There is ***exactly one*** person whom everybody loves".
   (c) In a complete bipartite graph $K_{m,n}$, if $m = n$, then $K_{m,n}$ have a Eulerian circuit.
   (d) In a simple graph $G = (V, E)$, if $|V| = 21$ and $|E| = 190$, then G is connected.
   (e) If A is an uncountable set and B is a countable set, then $A - B$ is uncountable.

6. Determine whether each of these functions is $O(x^2)$. Justify your answers.
   (a) $f(x) = x \log x$ $(2pts)$
   (b) $f(x) = x^4 / 2$ $(2pts)$
   (c) $f(x) = 2^x$ $(2pts)$
   (d) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$ $(2pts)$

7. Illustrate the operations of the following sorting algorithms:
   (a) Bucket sort on the array $A = \langle .79, .13, .16, .64, .39, .20, .89, .53, .71, .42 \rangle$. $(5pts)$
   (b) Counting sort on the array $B = \langle 7, 1, 3, 1, 2, 4, 5, 7, 2, 4, 3 \rangle$. $(5pts)$

8. Draw the complete binary search tree of height 3 on the keys {1, 2, …, 15}. Add the NIL leaves and color the nodes in three different ways such that the black-heights of the resulting red-black trees are 2, 3, and 4. $(10pts)$

9. A parking lot has 31 visitor spaces, numbered from 0 to 30. Visitors are assigned parking spaces using the hashing function $h(k) = k \bmod 31$, where k is the number formed from the first three digits on a visitor's license plate.
   (a) Which spaces are assigned by the hashing function to cars that have these first three digits on their license plates? $(5pts)$
   317, 918, 007, 100, 111, 310
   (b) Describe a procedure visitors should follow to find a free parking space, when the space they are assigned is occupied. $(7pts)$

10. Find the ordered rooted tree representing the compound proposition $(\neg(p \wedge q)) \leftrightarrow (\neg p \vee \neg q)$. Then use this rooted tree to find the prefix, postfix, and infix forms of this expression. $(10pts)$