

國 立 清 華 大 學 命 題 紙

96 學年度 通訊工程研究 系(所) 乙 組碩士班入學考試

科目 基礎計算機科學 科目代碼 2002 共 3 頁第 1 頁 *請在【答案卷卡】內作答

1. (5%) Using mathematical induction, prove that for all $n > 0$

$$\sum_{i=0}^n ((i+2) \times 2^i) = (n+1) \times 2^{n+1}$$

2. (3%) (a) Let A, B, C be three subsets of a universe U . Represent on a Venn diagram the set:

$$(A \cup B \cup C) \cap \overline{(A \cap B \cap C)}$$

- (3%) (b) In a group of n boxes, 10 are red, 5 are square, and 12 are square or red (or both). How many boxes are square but not red?

3. (4%) How many isomorphic unrooted trees are there with five vertices.
4. (20%) Consider a cycle of length n with vertices v_1, v_2, \dots, v_n . v_i is adjacent to v_{i+1} for $1 \leq i \leq n-1$ and v_n is adjacent to v_1 . We would like to assign one of k colors to each vertex in the cycle such that adjacent vertices get different colors. Find the number of ways of doing this (as a function of n and k).
5. The 26 letters A, B, C, \dots, Z are used to form strings of length n (n is some given positive integer).
- (a) (5%) how many strings containing the letter A can be formed?
- (b) (5%) how many strings can be formed if we do not allow repetitions?
- (c) (5%) how many strings sorted in alphabetical order can be formed (repetitions allowed)?
6. (10%) Demonstrate the insertion of the keys 5, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be $h(k) = k \bmod 9$.

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7. (10%) Suppose that we have numbers between 1 and 1000 in a binary search tree and want to search for the number 363. Which of the following sequences could *not* be the sequence of nodes examined? Please explain your answer.

- a 2, 252, 401, 398, 330, 344, 397, 363.
- b 924, 220, 911, 244, 898, 258, 362, 363.
- c 925, 202, 911, 240, 912, 245, 363.
- d 2, 399, 387, 219, 266, 382, 381, 278, 363.
- e 935, 278, 347, 621, 299, 392, 358, 363.

8. (10%) The *incidence matrix* of a directed graph $G = (V, E)$ is a $|V| \times |E|$ matrix $B=(b_{ij})$ such that

$$b_{ij} = \begin{cases} -1 & \text{if edge } j \text{ leaves vertex } i, \\ 1 & \text{if edge } j \text{ enters vertex } i, \\ 0 & \text{otherwise.} \end{cases}$$

Describe what the entries of the matrix product BB^T represent, where B^T is the transpose of B .

9. (10%) Dijkstra's algorithm for the single-source shortest-paths problem is as follows.

Dijkstra(G, w, s)

- 1 Initialize-Single-Source(G, s)
- 2 $S \leftarrow \emptyset$
- 3 $Q \leftarrow V[G]$
- 4 **while** $Q \neq \emptyset$
- 5 **do** $u \leftarrow \text{Extract-Min}(Q)$
- 6 $S \leftarrow S \cup \{u\}$

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7         for each vertex  $v$  in  $Adj[u]$ 
8         do Relax( $u, v, w$ )
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Suppose we change line 4 of Dijkstra's algorithm to the following.

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4 while  $|Q| > 1$ 
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This change causes the **while** loop to execute $|V|-1$ times instead of $|V|$ times. Is this proposed algorithm correct? Please explain your answer.

10. (10%) Draw a picture of the sequence $\langle 13, 4, 8, 19, 5, 11 \rangle$ stored as a doubly linked list using the single-array representation and multiple-array representation, respectively.