1. (20%) Please answer the following questions with explanations

(a) When we pass a baseband signal sample to a quantizer, often the compression law of either μ-law or A-law is used. What is the purpose of using the compressor? (4%)

(b) Consider a binary digital transmission system in which the transmitted signal is modulated with FSK as

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos(2\pi f_i t), 0 \le t \le T, i = 1, 2 \qquad (1)$$

What is the basic difference between coherent binary FSK and non-coherent binary FSK (i.e. what does coherent or non-coherent mean)? Please draw the block diagram of the binary FSK receiver for coherent and non-coherent binary FSK respectively. (4%)

(c) Considering the coherent binary FSK system, as defined in Eq. (1), symbols of 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in orthogonal frequencies. What is the frequency spacing of f_1 and f_2 for achieving the best spectral efficiency utilization? (4%)

(d) What is the desired autocorrelation property of the pseudo-random sequence for the application in the direct sequence spread spectrum communication? (4%)

(e) Two passband data transmission systems are to be compared. One system uses 16-PSK and the other uses 16-QAM for modulation. Both systems are required to produce the same symbol error rate. Please compare the signal-to-noise ratio requirement of these two systems. (4%)

2. (15%) Consider a baseband binary data transmission system as shown in Fig. A.

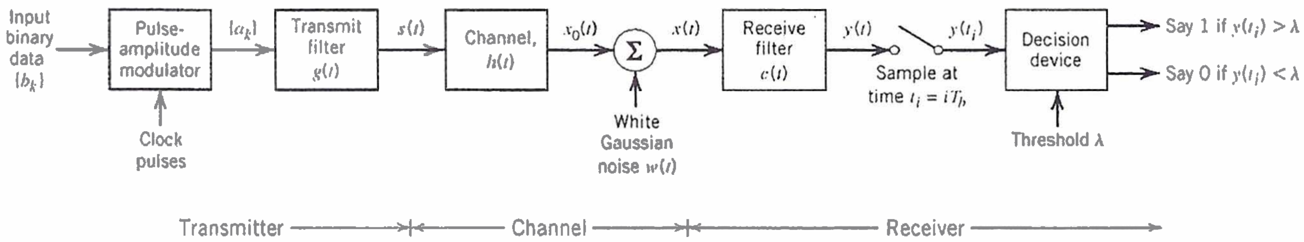

Figure A. Baseband data transmission system

The incoming binary system sequence $\{b_k\}$ consists of symbols 1 and 0, each of duration $T_b = 0.5\mu s$. The PAM modifies $\{b_k\}$ into $\{a_k\}$, where

$$a_k = \begin{cases} +1, & \text{if } b_k = 1 \\ -1, & \text{if } b_k = 0 \end{cases}$$

The receiver filter output is

$$y(t) = \sum_k a_k p(t - kT_b) + n(t)$$

where

$$p(t) = g(t) * h(t) * c(t)$$

(Hint: The Fourier series transform pair: $\sum_{k=-\infty}^{\infty} p(t-kT_b) \overset{F.T.}{\longleftrightarrow} \frac{1}{T_b}\sum_{n=-\infty}^{\infty} P\left(f-\frac{n}{T_b}\right)$)

(a) Please derive and find the condition such that the decision device input $y(t_i)$ is ISI (inter-symbol-interference) free. (10%)

(b) Let the frequency response of $p(t)$ be in the form of a rectangular function. Please find the highest frequency component of $P(f)$. (5%)

3. (20%) A random process defined by

$$X(t) = A(t)\cos(2\pi f_c t + \Theta),$$

is applied to an integrator producing the output

$$Y(t) = \int_{t-T}^{t} X(\tau)d\tau$$

(a) Suppose that f_c is a constant, $A(t)$ is a wide-sense stationary random process independent of Θ, and Θ is a random variable uniformly distributed in $[0, 2\pi]$. We denote the power spectral density of $A(t)$ by $S_A(f)$. Show that the power spectral density $S_Y(t)$ of $Y(t)$ is given by

$$S_Y(f) = \frac{1}{4}[S_A(f-f_c) + S_A(f+f_c)]T^2\text{sinc}^2(Tf)$$

(8%)

(b) Suppose that f_c is a constant, $A(t) = A$, where A is a Gaussian distributed random variable of zero mean and variance σ_A^2, and $\Theta = 0$. Determine the probability density function of the output $Y(t)$ at a particular time t_k. (4%)

(c) Based on the assumption in (b), is $Y(t)$ stationary? Give your reason. (4%)

(d) Based on the assumption in (b), is $Y(t)$ ergodic? Give your reason. (4%)

4. (15%) In a binary antipodal signalling scheme, the signals are given by

$$s_1(t) = -s_2(t) = \begin{cases} 2At/T, & 0 \le t \le T/2 \\ 2A(1-t/T), & T/2 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

The channel is AWGN, and its power spectral density is $N_0/2$. The two signals $s_1(t)$ and $s_2(t)$ have prior probabilities p_1 and $p_2 = 1 - p_1$, respectively.

(a) What is the impulse response of the matched filter? (2%)

(b) Given that $s_1(t)$ was sent, what is the probability density function of the output of the matched filter? (4%)

(c) Let the threshold of the decision device be λ. Express the average error probability in terms of λ. (4%)

(d) What is the optimal value of λ yielding the minimum average error probability? (5%)

5. (20%) In this problem, we consider the Alamouti code used for multi-input and multi-output (MIMO) fading channels. Two transmit antennas and one receive antenna are considered in this problem. Let us assume an M-ary (M-PSK or M-QAM) modulation scheme is used. In the Alamouti encoder, each group of m information bits is first modulated, where $m = \log_2 M$. Then, the encoder takes a block of two modulated symbols x_1 and x_2 in each encoding operation and maps them to the transmit antennas. A codeword transmits across two consecutive symbol transmission periods. Let us denote the transmit sequences from antennas one and two are given by $\mathbf{x}^1 = [x_1, -x_2^*]$ and $\mathbf{x}^2 = [x_2, x_1^*]$, respectively, where $*$ denotes the complex conjugate operation.

(a) Show that the transmit sequences from the two transmit antennas are orthogonal. (1%)

(b) Let the fading coefficients from antennas one and two be h_1 and h_2, respectively. Assume these fading coefficients are constant across two consecutive symbol transmission periods. At the receive antenna, the received signal over two consecutive symbol periods, denoted by r_1 and r_2, can be expressed as $r_1 = h_1x_1 + h_2x_2 + n_1$; $r_2 = h_2x_1^* - h_1x_2^* + n_2$, where n_1 and n_2 are independent complex Gaussian random variables with zero mean and power spectral density $N_0/2$ per dimension, representing the additive white Gaussian noise. For $i = 1, 2$, write $n_i = n_{i,x} + jn_{i,y}$, where $n_{i,x}$ and $n_{i,y}$ are independent real Gaussian random variables with zero mean and variance σ^2 and $j = \sqrt{-1}$. Assume that the channel fading coefficients h_1 and h_2, can be perfectly recovered at the receiver. Then, the decoder chooses a pair of signal $[\hat{x}_1, \hat{x}_2]$ from the signal constellation to minimize the distance metric $|r_1 - (h_1\hat{x}_1 + h_2\hat{x}_2)|^2 + |r_2 - (h_2\hat{x}_1^* - h_1\hat{x}_2^*)|^2$ over all possible $[\hat{x}_1, \hat{x}_2]$. Show that the decision rule becomes
$\hat{x}_1 = \arg\ \min_{\hat{x}_1 \in S}(|h_1|^2 + |h_2|^2 - 1)|\hat{x}_1|^2 + d^2(\tilde{x}_1, \hat{x}_1)$
$\hat{x}_2 = \arg\ \min_{\hat{x}_2 \in S}(|h_1|^2 + |h_2|^2 - 1)|\hat{x}_2|^2 + d^2(\tilde{x}_2, \hat{x}_2)$,
where $\tilde{x}_1 = h_1^*r_1 + h_2r_2^*$, $\tilde{x}_2 = h_2^*r_1 - h_1r_2^*$, $d^2(\tilde{x}_i, \hat{x}_i) = |\tilde{x}_i - \hat{x}_i|^2, i = 1, 2$, and S is the set of all possible modulated signal points in an M-ary signal constellation. (10%)

(c) Show that the decision rule in (b) becomes
$\hat{x}_1 = \arg\ \min_{\hat{x}_1 \in S} d^2(\tilde{x}_1, \hat{x}_1)$
$\hat{x}_2 = \arg\ \min_{\hat{x}_2 \in S} d^2(\tilde{x}_2, \hat{x}_2)$,
if M-PSK constellations are used. (1%)

(d) Show that the decoder used in (b) is a maximum likelihood decoder. (8%)

6. (10%) In this problem, we consider differential PSK (DPSK). Let $m(k)$ be a message data stream, where k is the sample time index. Let the sequence of encoded bits, $c(k)$, be encoded by using the following way: $c(k) = \overline{c(k-1) \oplus m(k)}$, where the symbol $\oplus$ represents modulo-2 addition and the overbar denotes complement.

   (a) Show that the first bit of the code bit sequence, $c(k=0)$, can be chosen arbitrarily without affecting the detected message sequence at receiver. (1%)

   (b) Please draw the block diagrams for DPSK transmitter and DPSK receiver. (3%)

   (c) Please describe the decision rules in your DPSK receiver. Please also give the reasons of choosing such decision rules. (6%)