

1. Let $L(x,y)$ be the statement “ x loves y ,” where the universe of discourse for both x and y consists of all people in the world. Use quantifiers to express the following statements.
 - (a) (3%) There is exactly one person whom everybody loves.
 - (b) (3%) There is someone who loves no one besides himself or herself.
2. (6%) A **palindrome** is a string whose reversal is identical to the string. How many bit strings of length n are palindromes?
3. (5%) Find the least number of cables required to connect 100 computers to 20 printers to guarantee that 20 computers can directly access 20 different printers. Justify your answer.
4. (8%) Find the solution of the recurrence relation $a_n = 4a_{n-1} - 3a_{n-2} + 2^n + n + 3$ with $a_0 = 1$ and $a_1 = 4$.
5. (5%) Find the coefficient of x^{10} in the power series of $x^4 / (1-3x)^3$.
6. How many relations are there on a set with n elements that are
 - (a) (5%) reflexive and symmetric?
 - (b) (5%) neither reflexive nor irreflexive?
7. (5%) How many nonisomorphic unrooted trees are there with five vertices?
8. (5%) What is wrong with this “proof” by strong induction?
 “Theorem” For every nonnegative integer n , $5n = 0$.
 Basis Step: $5 \cdot 0 = 0$.
 Inductive Step: Suppose that $5j = 0$ for all nonnegative integers j with $0 \leq j \leq k$. Write $k+1 = i+j$, where i and j are natural numbers less than $k+1$. By the induction hypothesis, $5(k+1) = 5(i+j) = 5i + 5j = 0 + 0 = 0$.
9. (4%) Convert the following expressions from infix to prefix and postfix:
 $A + B * C / D$
10. (8%) Write the status of the list $L = (12, 2, 16, 30, 8, 28, 4, 10, 20, 6, 18)$ after each phase of *heap sort*.
11. (7%) What is *static hashing*? What is *dynamic hashing*?
12. (7%) Give the formal definition and properties of a *B-tree*.
13. (8%) If we have n records with integer keys in the range $[0, n^2)$, then they may be sorted in $O(n \log n)$ time using heap or merge sort. Radix sort on a single key, i.e., $d = 1$ and $r = n^2$ takes $O(n^2)$ time. Show how to interpret the keys as 2 subkeys so that radix sort will take only $O(n)$ time to sort n records.

國立清華大學命題紙

95 學年度 通訊工程研究 系(所) 乙 組碩士班入學考試

科目 基礎計算機科學 科目代碼 2502 共 2 頁第 2 頁 *請在【答案卷卡】內作答

14. (8%) Prove that when algorithm *Depth First Search (DFS)* is applied to a connected graph, the edges of T form a tree.
15. (8%) Find the shortest path tree from every node to node 1 for the following graph by using the Dijkstra algorithm.

