1. (5%) What is the total number of nodes for a complete binary tree with level *N*?
   (5%) What is the minimum number of nodes for a binary tree with level *N*?

2. Explain the operation of following items. (3%) STACK. (3%) FIFO. (6%)Give one application for STACKS and one application for FIFO in a computer program.

3. Describe the procedure of sorting sequence 8, 3, 2, 4, 9, 7, 1 in ascending order using (9%) bubble sort. (9%) quicksort.

4. (5%) Draw the binary tree with the following inorder traversal of the tree.
   Inorder: JAPCHEDBTL

5. (5%) Show how to use a circular linked list to construct a page with *N* address.

6. (10%) Let the universe of discourse be the set of married persons, which is assumed to be the set of all integers. Determine whether each of the following assertions is true or false. (Note: you will get 2 points for each right answer, and be deducted 2 points for each wrong answer. The minimum point for this problem set is 0.)
   (a) $\forall x\, \exists y[x \text{ is married to } y]$
   (b) $\forall x\, \exists y[x + y = 0]$
   (c) $\forall x\, \forall y\, \exists! z[x + y = z]$
   (d) $\forall y\, \exists! x[x * y = 0]$
   (e) $\forall y\, \exists! x[x + y < 0]$

7. (5%) For each of the following functions, determine whether the function is injective, surjective, or bijective. You must briefly explain how do you get the answer. (**N** denotes the set of natural numbers. **I** denotes the set of all integers.)
   (a) $f: \mathbf{N} \rightarrow \mathbf{N} \times \mathbf{N}$
   $f(n) = \langle n, n + 1 \rangle$
   (b) $f: \mathbf{I} \rightarrow \mathbf{N}$
   $f(x) = |x|$

8. (10%) Let R_1 and R_2 be relations on a set $A = \{a, b, c, d\}$ where
   $R_1 = \{\langle a, a\rangle, \langle a, b\rangle, \langle b, d\rangle\}$
   $R_2 = \{\langle a, d\rangle, \langle b, c\rangle, \langle b, d\rangle, \langle c, b\rangle\}$
   Find R_1R_2, R_2R_1, R_1^2, R_2^3.

9. (5%) Give the generating function for the sequence $<2, 5, 13, 35, \ldots> = <2^n + 3^n>$ in closed form.

10. (10%) Solve the recurrence

$a_0 = 1/2$

$a_{n+1} = 2a_n, \quad$ for $n \geq 0$

by using a generating function.

11. (10%) Let M be the finite state machine of the following figure. The transition from a state to a state is represented by drawing a directed arc and labeling the arc with x, y, where x is the input and y is the output. For states s_i, s_j, $0 \leq i, j \leq 2$, let φ_{ij} denote the set of all output strings that M can produce as it goes from state s_i to state s_j. If $i = 2, j = 0$, for example, $\varphi_{20} = \{0\}(\{00\}^*\{1\}^*)^*$. An asterisk as an exponent is used to indicate that the substring is to be repeated any number of times (including zero). Find φ_{02}, φ_{22}.