1. Construct Venn diagrams for the following:

   (a) $\overline{(A \cup B)} \cap C$ (2%)

   (b) $(A \cup C) \cap B$ (2%)

2. The following formulae define functions from Z to Z, where Z denotes a set of objects comprising the positive and negative integers, and zero. Which of them are surjections, which are injections, and which are bijections?

   (a) $f(x) = x^2$ (4%)

   (b) $g(x) = 2x$ (4%)

   (c) $h(x) = x + 2$ (4%)

3. There are 73 faculty members in the College of Electrical Engineering and Computer Science at the National Tsing Hua University. Among them a total of 52 can play the piano, 25 can play the violin, and 10 can play the flute; 17 can play both piano and violin, 12 can play piano and flute, and 7 can play violin and flute; but only the Director of Institute of Communications Engineering can play all three instruments. How many of the faculty cannot play any of them? (7%)

4. How many ways can we obtain distinct arrangements of the letters of the word MISSISSIPPI? (7%)

5. (10%) Use the Prim's algorithm to find a minimal spanning tree for the weighted graph $G=(V, E)$ in the following figure. Let vertex a be the first vertex considered.
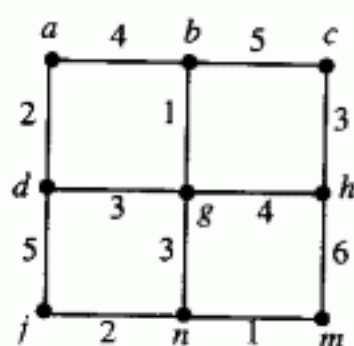
6. (10%) Compare the Dijkstra and Floyd-Warshall algorithms for finding shortest paths in weighted graphs.

7. (10%) Describe the items that are the same for two isomorphic graphs $G=(V_1, E_1)$ and $H=(V_2, E_2)$.

8. In a telephone survey of 1000 adults, respondents were asked about the expense of a college education and the relative necessity of some form of financial assistance. The respondents were classified according to whether they currently had a child in college and whether they thought the loan burden for most college students is too high, the right amount, or little. The proportions responding in each category are shown in the probability table in Table 1. Suppose one respondent is chosen at random from this group.

	Too High (A)	Right Amount (B)	Too little (C)
Child in College (D)	0.35	0.08	0.01
No Child in College (E)	0.25	0.20	0.11

Table 1: Probability table

(a) (5%) What is the probability that the respondent has a child in college?

(b) (5%) Given that the respondent has a child in college, what is the probability that he or she ranks the loan burden as "too high"?

(c) (5%) Are events D and A independent?

9. (5%) Over a long period of time it has been observed that a given rifleman can hit a target on a single trial with probability equal to 0.8. suppose he fires four shots at the target. What is the probability that he will hit the target at least once?

10. (10%) Random variable X has the following pdf (where $\lambda > 0$):

$$f(x) = \begin{cases} \lambda x^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(X)$.

11. (10%) The probability that a patient recovers from a rare blood disease is 0.4. If 100 people are known to have contracted this disease, what is the probability that less than 30 survive? Use Central Limit Theorem to approximate this probability in terms of $\Phi(z)$. where

$$\Phi(z) \overset{\text{def}}{=} \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2} \, dw.$$