

國立清華大學 命題紙

九十一學年度 電機工程學 通訊工程研究所 乙 組碩士班研究生招生考試

科目 通訊系統 科號 2404 3002 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. Consider a communication system where three equiprobable messages m_1 , m_2 , and m_3 are transmitted. Let m_1 , m_2 , and m_3 be encoded by signals $s_1(t)$, $s_2(t)$, and $s_3(t)$, respectively, given by

$$\begin{aligned} s_1(t) &= 3\sqrt{2} \cos 2\pi t \\ s_2(t) &= 2\sqrt{2} \sin 2\pi t \\ s_3(t) &= -2\sqrt{2} \sin 2\pi t \end{aligned}$$

where the signal duration is $0 \leq t \leq 1$ and each signal is zero outside this interval. Assume that the signals are transmitted over an additive white Gaussian noise channel.

- Find a set of orthonormal basis functions to represent the set of signals, and then draw the corresponding signal constellation. (5%)
 - Determine the optimum decision regions. (5%)
 - Determine an equivalent minimum-energy signal set that would yield the same probability of error as the signal set described above. Draw the corresponding signal constellation and optimum decision regions. (5%)
2. Consider a coherent binary frequency-shift keying (FSK) system where symbols 1 and 0 occur with equal probability. Let symbols 1 and 0 be encoded by signals $s_1(t)$ and $s_2(t)$, respectively, given by

$$s_i(t) = \begin{cases} \sqrt{2E_b/T_b} \cos(2\pi f_i t), & 0 \leq t \leq T_b \\ 0, & \text{elsewhere} \end{cases}$$

where $i=1, 2$, E_b is the transmitted signal energy per bit, T_b is the symbol duration, and $f_i = (n_c + i)/T_b$ for some fixed integer n_c . Assume that a white Gaussian noise process of zero mean and power spectral density $N_0/2$ is added during the transmission of an FSK signal.

- Determine the optimum receiver. (6%)
- Derive the error probability of the optimum receiver in terms of the complementary error function defined by

$$\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} \exp(-z^2) dz. \quad (9\%)$$

(You must give derivations, or you would get no points in this sub-problem!)

國立清華大學命題紙

九十一學年度 電機工程學系 通訊工程研究所 乙甲 組碩士班研究生招生考試

科目 通訊系統 科號 2404 3002 共 3 頁第 2 頁 *請在試卷【答案卷】內作答

3. Consider the following random-phase sinusoidal process

$$x(t) = A \cos(\omega_0 t + \theta), -\infty < t < \infty$$

where ω_0 is a constant, θ is a random variable uniformly distributed over $[0, 2\pi)$ and A is a binary random variable with probabilities $P_r[A=1] = p$ and $P_r[A=2] = 1-p$.

- (a) Find the mean and correlation function of $x(t)$. (10%)
 (b) Hilbert transformer is a linear time-invariant system with frequency response

$$H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \end{cases}$$

Assume that $x(t)$ is input to the Hilbert transformer and $y(t)$ is the associated output. Find the power spectral density of $y(t)$. (5%)

4. Assume that

$$x(t) = as(t) + n(t), -\infty < t < \infty$$

where $n(t)$ is white Gaussian noise with zero mean and power spectral density $S_n(f) = 1$, and the waveform of the signal $s(t)$ is given by

$$s(t) = \begin{cases} 1-t, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Find the matched filter impulse response and peak output signal squared to output noise variance. (8%)
 (b) Assume that $a=1$ or $a=-1$ with equal prior probability and that $y(t_0)$ is the matched filter output with the peak signal squared to output noise variance. Find the probability of error P_E of the detector that decides $a=1$ if $y(t_0) > 0$ and $a=-1$ if $y(t_0) < 0$. (You can express P_E in terms of the Q-function $Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty \exp(-z^2/2) dz$ or the complementary error function $\text{erfc}(u)$) (7%)

5. For a linear modulation with the in-phase component $s_I(t) = \frac{1}{2}m(t)$ and the quadrature component $s_Q(t) = \frac{1}{2}\hat{m}(t)$ ($\hat{m}(t)$ = Hilbert transform of $m(t)$), please show that $s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$ is a single-sideband (SSB) with upper sideband transmitted signal. (10%)

國立清華大學命題紙

九十一學年度 電機工程學 通訊工程研究所 乙 組碩士班研究生招生考試

科目 通訊系統 科號 2404 3002 共 3 頁第 3 頁 *請在試卷【答案卷】內作答

6. Consider that a double sideband-suppressed carrier (DSB-SC) modulated signal $s(t) = A_c \cos(2\pi f_c t)m(t)$ is transmitted over an additive white Gaussian noise channel with power spectral density $N_0/2$, where $m(t)$ is the message signal with average message power P and message bandwidth W .
- (a) Find the average power of DSB-SC modulated signal $s(t)$ and the average noise power in the message bandwidth. (5 %)
 - (b) Find the output signal of a coherent detector. (5 %)
 - (c) Find the output signal-to-noise ratio. (5 %)
7. Assume that a linear time-invariant filter of impulse response $h(t)$ (frequency response $H(f)$) is driven by a stationary random process $X(t)$ (with power spectral density $S_X(f)$) and $Y(t)$ is the associated output.
- (a) Find the power spectral density $S_Y(f)$ of $Y(t)$. (10 %)
 - (b) Show that the power spectral density $S_X(f)$ is always nonnegative. (5 %)